



Date:

Registration number:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27

B.Sc. MATHEMATICS - VI SEMESTER

SEMESTER EXAMINATION: APRIL 2022

(Examination conducted in July 2022)

MT6215-MATHEMATICS-VIII

Time- 2 ½ hrs

Max Marks-70

This question paper contains TWO printed pages and THREE parts.

I. **Answer any FIVE of the following questions.** (5X2=10)

1. Show that $|z - 1|^2 + |z + 1|^2 = 4$ represents a unit circle.
2. Evaluate $\lim_{z \rightarrow i} \left(\frac{z^3 + i}{1 - zi} \right)$.
3. Define Harmonic function.
4. Evaluate $\int_0^{3+i} z^2 dz$ along the line $3y = x$.
5. Find the fixed points of the transformation $w = \frac{3z-4}{z}$.
6. Find the Laplace transform of $(t^2 - 1)$.
7. Find the inverse Laplace transform of $\left(\frac{2s^2 - 5s + 8}{s^3} \right)$.
8. State the Convolution theorem.

II. **Answer any SEVEN of the following questions.** (7×6=42)

9. Show that $\arg \left(\frac{z-i}{z+1} \right) = \frac{\pi}{2}$ represents a circle. Find its centre and radius.
10. If a complex function $f(z) = u + iv$ is analytic in a domain D then prove that the first order partial derivatives of u, v with respect to x, y exists and satisfy the C-R equations, $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$.
11. Show that $u = e^x \cos y + xy$ is harmonic and find its harmonic conjugate v .
12. Find the analytic function whose real part is $\left(r + \frac{1}{r} \right) \cos \theta$.

13. If a complex function $f(z) = u + iv$ is analytic and $f'(z)$ is continuous at each point within and on a closed contour, then prove that $\oint_c f(z)dz = 0$.

14. Evaluate $\oint_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where c is the circle $|z| = 4$.

15. Evaluate $\oint_c \frac{e^{2z}}{(z-2)^3} dz$ where c is the circle $|z| = 3$.

16. Discuss the transformation $w = \sin z$.

17. Find the bilinear transformation which maps $0, -i, -1$ in the z -plane onto $i, 1, 0$ in the w -plane.

III. **Answer any THREE of the following questions.**

(3X6=18)

18. Solve the system of equation by Gauss-Seidel method:

$$27x + 6y - z = 85$$

$$6x + 15y + 2z = 72$$

$$x + y + 54z = 110$$

19. Use Euler's modified method to compute y for $x = 0.05$, given that $\frac{dy}{dx} = x + y$ with the initial condition $x_0 = 0, y_0 = 1$ and $h = 0.05$.

20. Find the Laplace transform of e^{-t} for $0 \leq t \leq 2$ and $f(t+2) = f(t)$.

21. Find the inverse Laplace transform of the function $\left(\frac{s-3}{s^2+4s+13}\right)$.
