Register Number:

Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE –560 027

M.Sc. STATISTICS – IV SEMESTER

SEMESTER EXAMINATION – JULY 2022

ST 0120: Advanced Statistical Inference

Time: 2 ½ hrs **Max: 70 Marks**

This question paper has **TWO** printed pages and **TWO** sections

SECTION – A

I Answer any SIX of the following: 6x 3= 18

1. Define consistent estimator. State and prove a sufficient condition for a statistic to be consistent.
2. Show that Y_n which is the sample maximum is consistent for θ when samples are drawn from $U(0,\theta)$.
3. Show that Consistent asymptotically Normal (CAN) estimators are necessarily consistent.
4. State the conditions for a family of distributions belonging to Cramer's family.
5. Write a note on Bootstrapping technique.
6. Define the stopping rule in SPRT with an illustrative example.
7. Explain the terms parametric and nonparametric models with suitable examples.
8. Define Sign test. Mention its null distribution.

SECTION – B

II Answer any FOUR of the following: 4 x 13 = 52

9. A) Prove that marginal consistency and joint consistency are equivalent.
B) Define CAN and Best asymptotically Normal (BAN) estimators. Give example consistent but not CAN. (7+6)

10. A) State and prove the invariance property of CAN estimators.
B) Define the Asymptotic relative efficiency (ARE) of estimators. Suppose that $X{\sim}P(\lambda)$, based on the random sample $X_1, X_2, \ldots$ define estimator for P(X=0) and λ, obtain ARE of these estimators. (7+6)

11. A) Define M-estimators. Obtain asymptotic distribution of Huber estimator if $X_1, X_2, \ldots, X_n$ are from symmetric distribution, symmetric about zero.
B) With an algorithm explain how to compute the variance of an estimator obtained from a sample of size n using bootstrap technique. (8+5)

12. A) Obtain Sequential Probability Ratio Test (SPRT) for testing the parameter p, if the samples are drawn from $B(n,p)$.
B) Obtain stopping time bounds for SPRT with an illustration. (7+6)

13. A) State and prove Wald's fundamental identity.
    B) Define Wilcoxon-Mann-Whitney U-statistic for two-sample problem. Derive null distribution of Wilcoxon-Mann-Whitney U-statistic under the null hypothesis.
    (6+7)
14. A) Define Kolmogorov-Smirnov test for one-sample and derive its distribution under the null hypothesis.
    B) Describe Mood's test for two-sample scale problems. Obtain the null distribution of the statistic. (7+6)

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