Paper : **II**

Subject : **MATHEMATICAL SCIENCES**

Subject Code : **26**

Roll No. | | | | | | | | |

(Figures as per admission card)

OMR Sheet No. : ________________

BOOKLET SERIAL NO.

Name & Signature of Invigilator/s

Signature : ______________________________

Name : ______________________________

Time : 2 Hours Maximum Marks : 200

Number of Pages in this Booklet : **24** Number of Questions in this Booklet : **100**

ಅಭ್ಯರ್ಥಿಗಳಿಗೆ ಸೂಚನೆಗಳು

1. ಈ ಪುಟದ ಮೇಲ್ತುದಿಯಲ್ಲಿ ಒದಗಿಸಿದ ಸ್ಥಳದಲ್ಲಿ ನಿಮ್ಮ ರೋಲ್ ನಂಬರನ್ನು ಬರೆಯಿರಿ.
2. ಈ ಪತ್ರಿಕೆಯು ಬಹು ಆಯ್ಕೆ ವಿಧದ ನೂರು (100) ಪ್ರಶ್ನೆಗಳನ್ನು ಒಳಗೊಂಡಿದೆ.
3. ಪರೀಕ್ಷೆಯ ಪ್ರಾರಂಭದಲ್ಲಿ, ಪ್ರಶ್ನೆ ಪುಸ್ತಿಕೆಯನ್ನು ನಿಮಗೆ ನೀಡಲಾಗುವುದು. ಮೊದಲ 5 ನಿಮಿಷಗಳಲ್ಲಿ ನೀವು ಪುಸ್ತಿಕೆಯನ್ನು ತೆರೆಯಲು ಮತ್ತು ಕೆಳಗಿನಂತೆ ಕಡ್ಡಾಯವಾಗಿ ಪರೀಕ್ಷಿಸಲು ಕೋರಲಾಗಿದೆ.
   (i) ಪ್ರಶ್ನೆಪುಸ್ತಿಕೆಗೆ ಪ್ರವೇಶಾವಕಾಶ ಪಡೆಯಲು, ಈ ಹೊದಿಕೆ ಪುಟದ ಅಂಚಿನ ಮೇಲಿರುವ ಪೇಪರ್ ಸೀಲನ್ನು ಹರಿಯಿರಿ. ಸ್ಟಿಕ್ಕರ್ ಸೀಲ್ ಇಲ್ಲದ ಅಥವಾ ತೆರೆದ ಪುಸ್ತಿಕೆಯನ್ನು ಸ್ವೀಕರಿಸಬೇಡಿ.
   (ii) **ಪುಸ್ತಿಕೆಯಲ್ಲಿನ ಪ್ರಶ್ನೆಗಳ ಸಂಖ್ಯೆ ಮತ್ತು ಪುಟಗಳ ಸಂಖ್ಯೆಯನ್ನು ಮುಖಪುಟದ ಮೇಲೆ ಮುದ್ರಿಸಿದ ಮಾಹಿತಿಯೊಂದಿಗೆ ತಾಳೆ ನೋಡಿರಿ. ಪುಟಗಳು/ಪ್ರಶ್ನೆಗಳು ಕಾಣೆಯಾದ ಅಥವಾ ದ್ವಿಪ್ರತಿ ಅಥವಾ ಅನುಕ್ರಮವಾಗಿಲ್ಲದ ಅಥವಾ ಇತರ ಯಾವುದೇ ವ್ಯತ್ಯಾಸದ ದೋಷಪೂರಿತ ಪುಸ್ತಿಕೆಯನ್ನು ಕೂಡಲೆ 5 ನಿಮಿಷದ ಅವಧಿ ಒಳಗೆ, ಸಂವೀಕ್ಷಕರಿಂದ ಸರಿ ಇರುವ ಪುಸ್ತಿಕೆಗೆ ಬದಲಾಯಿಸಿಕೊಳ್ಳಬೇಕು. ಆ ಬಳಿಕ ಪ್ರಶ್ನೆಪತ್ರಿಕೆಯನ್ನು ಬದಲಾಯಿಸಲಾಗುವುದಿಲ್ಲ, ಯಾವುದೇ ಹೆಚ್ಚು ಸಮಯವನ್ನೂ ಕೊಡಲಾಗುವುದಿಲ್ಲ.**
4. ಪ್ರತಿಯೊಂದು ಪ್ರಶ್ನೆಗೂ (A), (B), (C) ಮತ್ತು (D) ಎಂದು ಗುರುತಿಸಿದ ನಾಲ್ಕು ಪರ್ಯಾಯ ಉತ್ತರಗಳಿವೆ. ನೀವು ಪ್ರಶ್ನೆಯ ಎದುರು ಸರಿಯಾದ ಉತ್ತರದ ಮೇಲೆ, ಕೆಳಗೆ ಕಾಣಿಸಿದಂತೆ ಅಂಡಾಕೃತಿಯನ್ನು ಕಪ್ಪಾಗಿಸಬೇಕು.

   ಉದಾಹರಣೆ : (A) (B) ● (D)

   (C) ಸರಿಯಾದ ಉತ್ತರವಾಗಿದ್ದಾಗ.
5. ಈ ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯ ಜೊತೆಯಲ್ಲಿ **ಕೊಟ್ಟಿರುವ OMR ಉತ್ತರ ಹಾಳೆಯಲ್ಲಿ ನಿಮ್ಮ ಉತ್ತರಗಳನ್ನು ಸೂಚಿಸತಕ್ಕದ್ದು**. OMR ಹಾಳೆಯಲ್ಲಿ ಅಂಡಾಕೃತಿಯಲ್ಲದೆ ಬೇರೆ ಯಾವುದೇ ಸ್ಥಳದಲ್ಲಿ ಉತ್ತರವನ್ನು ಗುರುತಿಸಿದರೆ, ಅದರ ಮೌಲ್ಯಮಾಪನ ಮಾಡಲಾಗುವುದಿಲ್ಲ.
6. OMR ಉತ್ತರ ಹಾಳೆಯಲ್ಲಿ ಕೊಟ್ಟ ಸೂಚನೆಗಳನ್ನು ಜಾಗರೂಕತೆಯಿಂದ ಓದಿರಿ.
7. ಎಲ್ಲಾ ಕರಡು ಕೆಲಸವನ್ನು ಪುಸ್ತಿಕೆಯ ಕೊನೆಯಲ್ಲಿ ಮಾಡತಕ್ಕದ್ದು.
8. ನಿಮ್ಮ ಗುರುತನ್ನು ಬಹಿರಂಗಪಡಿಸಬಹುದಾದ ನಿಮ್ಮ ಹೆಸರು ಅಥವಾ ಯಾವುದೇ ಚಿಹ್ನೆಯನ್ನು, ಸಂಗತವಾದ ಸ್ಥಳ ಹೊರತು ಪಡಿಸಿ, OMR ಉತ್ತರ ಹಾಳೆಯ ಯಾವುದೇ ಭಾಗದಲ್ಲಿ ಬರೆದರೆ, ನೀವು ಅನರ್ಹತೆಗೆ ಬಾಧ್ಯರಾಗುತ್ತೀರಿ.
9. ಪರೀಕ್ಷೆಯು ಮುಗಿದನಂತರ, ಕಡ್ಡಾಯವಾಗಿ OMR ಉತ್ತರ ಹಾಳೆಯನ್ನು ಸಂವೀಕ್ಷಕರಿಗೆ ನೀವು ಹಿಂತಿರುಗಿಸಬೇಕು ಮತ್ತು ಪರೀಕ್ಷಾ ಕೊಠಡಿಯ ಹೊರಗೆ OMRನ್ನು ನಿಮ್ಮೊಂದಿಗೆ ಕೊಂಡೊಯ್ಯಕೂಡದು.
10. ಪರೀಕ್ಷೆಯ ನಂತರ, ಪರೀಕ್ಷಾ ಪ್ರಶ್ನೆಪತ್ರಿಕೆಯನ್ನು ಮತ್ತು ನಕಲು OMR ಉತ್ತರ ಹಾಳೆಯನ್ನು ನಿಮ್ಮೊಂದಿಗೆ ತೆಗೆದುಕೊಂಡು ಹೋಗಬಹುದು.
11. **ನೀಲಿ/ಕಪ್ಪು ಬಾಲ್‌ಪಾಯಿಂಟ್ ಪೆನ್ ಮಾತ್ರವೇ ಉಪಯೋಗಿಸಿರಿ.**
12. **ಕ್ಯಾಲ್ಕುಲೇಟರ್, ವಿದ್ಯುನ್ಮಾನ ಉಪಕರಣ ಅಥವಾ ಲಾಗ್ ಟೇಬಲ್ ಇತ್ಯಾದಿಯ ಉಪಯೋಗವನ್ನು ನಿಷೇಧಿಸಲಾಗಿದೆ.**
13. **ಸರಿ ಅಲ್ಲದ ಉತ್ತರಗಳಿಗೆ ಋಣ ಅಂಕ ಇರುವುದಿಲ್ಲ.**
14. **ಕನ್ನಡ ಮತ್ತು ಇಂಗ್ಲೀಷ್ ಆವೃತ್ತಿಗಳ ಪ್ರಶ್ನೆಪತ್ರಿಕೆಗಳಲ್ಲಿ ಯಾವುದೇ ರೀತಿಯ ವ್ಯತ್ಯಾಸಗಳು ಕಂಡುಬಂದಲ್ಲಿ, ಇಂಗ್ಲೀಷ್ ಆವೃತ್ತಿಗಳಲ್ಲಿರುವುದೇ ಅಂತಿಮವೆಂದು ಪರಿಗಣಿಸಬೇಕು.**

Instructions for the Candidates

1. Write your roll number in the space provided on the top of this page.
2. This paper consists of Hundred multiple-choice type of questions.
3. At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as below :
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of the cover page. Do not accept a booklet without sticker seal or open booklet.
   (ii) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.**
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.

   Example : (A) (B) ● (D)

   where (C) is the correct response.
5. Your responses to the questions are to be indicated in the **OMR Sheet kept inside this Booklet.** If you mark at any place other than in the circles in the OMR Sheet, it will not be evaluated.
6. Read the instructions given in OMR carefully.
7. Rough Work is to be done in the end of this booklet.
8. If you write your name or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
9. You have to return the OMR Answer Sheet to the invigilators at the end of the examination compulsorily and must NOT carry it with you outside the Examination Hall.
10. You can take away question booklet and carbon copy of OMR Answer Sheet after the examination.
11. **Use only Blue/Black Ball point pen.**
12. **Use of any calculator, electronic gadgets or log table etc., is prohibited.**
13. **There is no negative marks for incorrect answers.**
14. **In case of any discrepancy found in the Kannada translation of a question booklet the question in English version shall be taken as final.**

MATHEMATICAL SCIENCES
Paper – II

Note : This paper contains **hundred (100)** objective type questions of **two (2)** marks **each**. **All** questions are **compulsory**.

1. The number of pairs of integers a and b satisfying $0 < a < b$ and $a^b = b^a$ is

(A) 0

(B) 1

(C) 2

(D) infinite

2. Let $\sum_{n=0}^{\infty} \frac{(2n)!(3n)!}{n!(4n)!}$ — (I) and

$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3n+2}{2n}}}$ — (II) then

(A) Series (I) converges and series (II) diverges

(B) Series (I) diverges and series (II) converges

(C) Both the series (I) and (II) converge

(D) Both the series (I) and (II) diverge

3. The series $\sum_{n=1}^{\infty} (-1)^n \frac{2n+1}{3n+5}$ is

(A) Divergent

(B) Convergent

(C) Conditionally convergent

(D) Absolutely convergent

4. The radius of convergence of the power series

$$\sum_{n=1}^{\infty} \left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)...\left(1+\frac{n}{n}\right)\right\} z^n \text{ is}$$

(A) $\frac{e}{4}$

(B) $\frac{4}{e}$

(C) 4e

(D) e^4

5. The set $[e, \pi] \cap Q$ is

(A) compact

(B) connected

(C) compact but not connected

(D) neither compact nor connected

6. Suppose that $g : \mathbb{R} \to \mathbb{R}$ is continuous on $\mathbb{R}$ and $g(x) = 0$, for every rational x. Then

(A) $g(\sqrt{7}) < 0$

(B) $g(\sqrt{7}) > 0$

(C) $g(\sqrt{7}) = 0$

(D) $g(\sqrt{7}) \neq 0$

7. Let $f_n : \mathbb{R} \to \mathbb{R}$ be differentiable for each $n = 1, 2, ...,$ with $|f_n'(x)| \leq 1$, for all n and x. Assume $\lim_{n\to\infty} f_n(x) = g(x)$. Then

(A) g is continuous for all x

(B) g is continuous only for x > 0

(C) g is continuous only for x < 0

(D) g is not continuous

8. The sum of the series

$$\frac{1^2}{1!} + \frac{1^2 + 2^2}{2!} + \frac{1^2 + 2^2 + 3^2}{3!} + ... \text{ is}$$

(A) $\frac{6}{e}$

(B) $\frac{6}{17e}$

(C) $\frac{17e}{6}$

(D) $\frac{6e}{17}$

9. The value of $\lim_{n\to\infty} \frac{n^{\sqrt{3}}}{\left(1+\sqrt{2}\right)^n}$ is

(A) 1

(B) $+\infty$

(C) 0

(D) $\sqrt{2}$

10. Which one of the following statements is false ?

(A) If f is bounded and has finitely many discontinuities on [a, b], then f is Riemann integrable on [a, b]

(B) If f is monotonic on [a, b], then f is Riemann integrable on [a, b]

(C) If f is continuous on [a, b], then f is Riemann integrable on [a, b]

(D) If f and g are not Riemann integrable on [a, b], then fg is not Riemann integrable on [a, b]

11. Let f : $\mathbb{R}^2 \to \mathbb{R}^1$ be defined by

$$f(x,y) = \frac{xy}{x^2+y^2}, \text{ if } (x, y) \neq (0, 0)$$

and f(0, 0) = 0. Which one of the following statement is true ?

(A) $D_1 f(0, 0) = 0$

(B) f is not continuous at (0, 0)

(C) $D_2 f(0, 0) = 0$

(D) f is continuous at (0, 0)

12. $\lim\limits_{n\to\infty} \frac{1}{n}\left(\frac{1}{2}+\frac{2}{3}+\ldots+\frac{n}{n+1}\right) =$

(A) 0

(B) ∞

(C) 1

(D) limit does not exist

13. Let R denote an arbitrary 3×4 matrix of rank 2 and O denote the 3×4 matrix all of whose entries are 0. What is the rank of the following 12×16 matrix ?

$$\begin{pmatrix} R & O & R & O \\ R & -R & R & -R \\ O & R & O & R \\ O & O & O & R \end{pmatrix}$$

(A) 2

(B) 4

(C) 6

(D) 8

14. Let A be a real 2×2 matrix such that $A^8 = I$ but $A^4 \neq I$, where I denotes the identity matrix of size 2×2. Then the trace of A equals

(A) $\pm\sqrt{2}$

(B) 0

(C) ± 1

(D) $\pm\frac{1}{\sqrt{2}}$

15. Let M be a real 3×3 matrix which has 0, 1 and -1 as eigenvalues. Which of the following is not true of M ?

(A) $M^8 + M^4 = M^6 + M^2$

(B) $M^7 + 3M^2 + 2M = M^6 + 2M^4 + 3M^3$

(C) $M^8 + 3M^2 + 2M = M^6 + 2M^4 + 3M^3$

(D) $M^9 + 3M^2 + 2M = M^6 + 2M^4 + 3M^3$

16. A 4×4 real matrix has rank 3. What is the rank of its adjoint ?

(A) 3

(B) 1

(C) 2

(D) cannot be determined from the given data

17. How many different (non-equivalent) non-degenerate symmetric bilinear forms are there on $\mathbb{R}^4$?

(A) 2

(B) 5

(C) 4

(D) infinitely many

18. The matrix $\begin{pmatrix} 2 & 1 \\ 1 & x \end{pmatrix}$ is positive definite if and only if

(A) $x > 0$

(B) $x > \frac{1}{2}$

(C) $x > \frac{1}{3}$

(D) $x > 2$

19. Let V be the vector space of all $n \times n$ real skew-symmetric matrices. Then $\dim_{\mathbb{R}} V$ is

(A) $\frac{1}{2}n(n+1)$

(B) $n^2 - 1$

(C) $\frac{n}{2}$

(D) $\frac{1}{2}(n-1)n$

20. Let $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$. Choose the correct statement in the following.

(A) A is not diagonalizable

(B) A is diagonalizable and its diagonal form is $\begin{pmatrix} 3 & 0 \\ 0 & -1 \end{pmatrix}$

(C) A is similar to $\begin{pmatrix} -3 & 0 \\ 0 & 1 \end{pmatrix}$

(D) A is similar to $\begin{pmatrix} -1 & 0 \\ 0 & -3 \end{pmatrix}$

21. Let x, y, z be linearly independent vectors in $\mathbb{R}^4$. Then, the vectors x + y, y + z and z + x are

(A) linearly independent

(B) linearly dependent

(C) linearly independent only if x, y, z are pairwise orthogonal

(D) will span a 2-dimensional subspace of $\mathbb{R}^4$

22. The dimension of the subspace W of $\mathbb{R}^n$, where W is given by $W = \{(x_1, x_2,...,x_n)|x_i \in \mathbb{R}, 1 \leq i \leq n$ and $x_1 + x_2 + ... + x_n = 0\}$ is

(A) 0

(B) n

(C) n – 1

(D) 2

23. Number of ways of selecting 30 elements from a set of 101 elements where order does not count and repetitions allowed is

(A) $\begin{pmatrix}101\\71\end{pmatrix}$

(B) $\begin{pmatrix}101\\70\end{pmatrix}$

(C) $\begin{pmatrix}130\\30\end{pmatrix}$

(D) $\begin{pmatrix}131\\71\end{pmatrix}$

24. If p is prime number and g is a non-zero element of the field $\mathbb{F}_p$ with p elements, then the order of g in the multiplicative group, $\mathbb{F}_p \setminus \{0\}$

(A) is always p

(B) is always less than p – 1

(C) is always greater than 1

(D) can be less than p – 1

25. The number of generators of the cyclic group of order 30 is

(A) 20 (B) 8

(C) 1 (D) 15

26. If R is a ring of cardinality 25 and with a multiplicative unit, then

(A) R may not contain a field

(B) R contains an ideal not equal to (0) and not equal to R

(C) The only ideals of $\mathbb{R}$ are (0) and R

(D) R contains a field with 5 elements

27. If R is a unique factorization domain, then

(A) R is a principal ideal domain

(B) R is a Euclidean domain

(C) Any sub ring of R is a unique factorization domain

(D) R may have a non-zero prime ideal that is not maximal

28. The number of monic irreducible polynomials of degree 2 over the field $\mathbb{F}_7$ of 7 elements is

(A) 21 (B) 28

(C) 42 (D) 49

29. The polynomial

$f(X) = X^4 + X^3 + X^2 + X + 6$ is

(A) irreducible over the field of rational numbers

(B) irreducible over the field of real numbers

(C) product of two irreducible polynomials over the field of rational numbers

(D) product of three irreducible polynomials over the field of rational numbers

30. In the symmetric group S_4 on four symbols, the number of elements of order exactly 4 is

(A) 8

(B) 3

(C) 1

(D) 6

31. In the polynomial ring $\mathbb{R}[x]$ over real numbers, which one of the following is true ?

(A) Every ideal in $\mathbb{R}[x]$ is generated by an element of degree 1

(B) Every ideal in $\mathbb{R}[x]$ is generated by an element of degree ≤ 2

(C) Every prime ideal in $\mathbb{R}[x]$ is generated by an element of degree ≤ 2

(D) Every maximal ideal in $\mathbb{R}[x]$ is generated by an element of degree 1

32. Which one of the following quotient rings is a field ?

(A) $\mathbb{F}_3[X] / X^2 + X + 1$, where $\mathbb{F}_3$ is the finite field with three elements

(B) $\mathbb{Z}[X] / (X - 3)$

(C) $Q\,[X] / X^2 + X + 1$

(D) $\mathbb{F}_2[X] / (X^3 + X^2 + X + 1)$, where $\mathbb{F}_2$ is the finite field with two elements

33. For $|z| < 1$, $\prod_{k=0}^{\infty} \sum_{m=0}^{9} z^{10^k m} =$

(A) $\frac{z}{1+z^2}$

(B) $\frac{z}{1-z^2}$

(C) $\frac{1}{1+z}$

(D) $\frac{1}{1-z}$

34. The series $\sum_{n=0}^{\infty}\left(\frac{z^n}{n!} + \frac{n^2}{z^n}\right)$, $(z \in \mathbb{C})$, converges for

(A) $|z| < 1$

(B) $|z| > 1$

(C) $|z| = 1$

(D) $z \neq 0$

35. $\frac{1}{2\pi i} \int_{|z|=2} \frac{e^z - 1}{z^2(z-1)}\, dz =$

(A) e

(B) e – 2

(C) e – 1

(D) 2e

36. Let $\mathbb{R}^3$ be given the usual topology. Consider the following subsets of $\mathbb{R}^3$.

$A = \{(x, y, z) \in \mathbb{R}^3 \mid xyz = 0\}$

$B = \{(x, y, z) \in \mathbb{R}^3 \mid xyz = 1\}$

$C = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}$

Then which one of the following statement is correct ?

(A) A and B are homeomorphic

(B) A and C are homeomorphic

(C) B and C are homeomorphic

(D) B and C are nonhomeomorphic

37. Let A and B be subsets of a topological space X. Then which one of the following need not be true ?

(A) $\overline{A \cup B} \subset \bar{A} \cup \bar{B}$

(B) $\overline{A \cap B} \subset \bar{A} \cap \bar{B}$

(C) $\overline{A - B} \subset \bar{A} - \bar{B}$

(D) $\bar{A} \cup \bar{B} \subset \overline{A \cup B}$

38. Which one of the following statement is correct ?

(A) Finite topological spaces are never connected

(B) Infinite set with finite complement topology is connected

(C) If X is connected and A is a proper subset of X then $BdA = \phi$

(D) A connected space is always path connected

39. Which one of the following statement is correct ?

(A) $\mathbb{R}^w$ in the box topology is metrizable

(B) $\mathbb{R}^w$ in the product topology is metrizable

(C) $\mathbb{R}^J$ in the product topology is metrizable

(D) $\mathbb{R}^n$ is not metrizable

40. The path components of the subspace $Y = (-1, \pi) \cup (e, 5) \cup (5, 8)$ of $\mathbb{R}$ are

(A) $(-1, \pi)$, $(e, 5)$ and $[5, 8)$

(B) $(-1, 5)$ and $(5, 8)$

(C) $(-1, \pi) \cup (e, 5) \cup [5, 8]$

(D) $[-1, 5]$ and $[5, 8]$

41. If f_1 and f_2 are linearly independent solutions of $y'' + p(x)\,y' + q(x)y = 0$, then

(A) $p(x) = \dfrac{f_1''f_2 - f_2''f_1}{f_2'f_1 - f_1'f_2}$ and

$q(x) = \dfrac{f_2''f_1' - f_1''f_2'}{f_1f_2' - f_2f_1'}$

(B) $p(x) = \dfrac{f_1'f_2 - f_2'f_1}{f_2'f_1 + f_1'f_2}$ and

$q(x) = \dfrac{f_2'f_1' + f_1''f_2}{f_1f_2' - f_2f_1'}$

(C) $p(x) = \dfrac{f_1'f_2'' - f_2'f_1''}{f_2'f_1 - f_1'f_2}$ and

$q(x) = \dfrac{f_2'f_1'' - f_1'f_2''}{f_1f_2' - f_2f_1'}$

(D) $p(x) = \dfrac{f_1'f_2 - f_2''f_1}{f_2f_1' - f_1f_2'}$ and

$q(x) = \dfrac{f_2f_1'' - f_1f_2''}{f_1'f_2 - f_2'f_1}$

42. For the initial value problem $y' = y^2 + 1$, $y(0) = 0$, the largest h such that, existence of the solution in $|x| \le h$, predicted by Picard's theorem is

(A) $1/4$

(B) $1/3$

(C) $1/2$

(D) 1

43. The general partial differential equation of second order for a function of two independent variables x and y

$$R\frac{\partial^2 z}{\partial x^2} + S\frac{\partial^2 z}{\partial x \partial y} + T\frac{\partial^2 z}{\partial y^2} + f(x, y, z, p, q) = 0$$

where R, S and T are continuous functions of x and y and possessing partial derivatives defined on some domain D on the xy-plane, then it is said to be

(A) Parabolic at a point (x, y) in D if $S^2 - 4RT > 0$

(B) Hyperbolic at a point (x, y) in D if $S^2 - 4RT = 0$

(C) Elliptic at a point (x, y) in D if $S^2 - 4RT < 0$

(D) Hyperbolic at a point (x, y) in D if $S^2 - 4RT < 0$

44. If $u = u(x, y)$ is a solution of the Cauchy's problem $\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial y} = 6x$, $u(0, y) = 3y$, then the value of $u(1, 1)$ is

(A) 2

(B) 3

(C) 1

(D) 4

45. The sufficient condition for the convergence of Newton-Raphson iteration scheme $a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$, is

(A) $|f(a_n)f'(a_n)| < |f''(a_n)|$

(B) $\left|\frac{f(a_n)f''(a_n)}{[f''(a_n)]^2}\right| > 1$

(C) $\left|\frac{f(a_n)f'(a_n)}{[f''(a_n)]^2}\right| < 1$

(D) $\left|\frac{f(a_n)f''(a_n)}{[f'(a_n)]^2}\right| < 1$

46. Given the following data

x	1	2	4	8
f(x)	3	7	21	73

,

the piecewise linear interpolating polynomial for $1 \le x \le 2$ is

(A) $p(x) = 4x + 1$

(B) $p(x) = 4x - 1$

(C) $p(x) = 6x - 3$

(D) $p(x) = 2x + 3$

47. The functional

$J[y] = \int_0^2 (t^2 + y'^2 - 2ty')dt$ with $y(0) = 0$ and $y(2) = 3$ has

(A) no solution

(B) an infinite number of solutions

(C) exactly two solutions

(D) a unique solution

48. If $y(t) = t + \int_0^{1/2} y(s)ds$, then which of the following statement is true ?

(A) The iterated kernels are $\left(\frac{1}{3^{n-1}}\right)$, n = 1, 2, 3,......

(B) The resolvent kernel is 3

(C) $y = t + \frac{1}{4}$

(D) $y(t) = t + \int_0^{1/2} s\,ds$

49. The solution of the Initial value problem

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x,$$

$y(0) = 2$, $y'(0) = 4$, is

(A) $y = \frac{3}{2}e^{3x} + 2e^{-x} - e^x + 2\cos x - \sin x$

(B) $y = \frac{3}{2}e^{3x} + 2e^{-x} - 2e^x + 2\sin x - \cos x$

(C) $y = \frac{3}{2}e^{3x} + 2e^{-x} - \frac{1}{2}e^x + 2\sin x - \cos x$

(D) $y = \frac{3}{2}e^{3x} + \frac{1}{2}e^{-x} - 2e^x + 2\sin x - \cos x$

50. The eigenvalues of the Sturm – Liouville problem are

(A) Complex with negative imaginary part

(B) Complex with positive imaginary part

(C) Real and negative

(D) Real and non-negative

51. The Green's function for the boundary value problem $\frac{d^2y}{dx} + \lambda y = x$,

$y(0) = y\left(\pi/2\right) = 0$ is

(A) $G(x,t) = \begin{cases} x\left(1 - \frac{2t}{\pi}\right) ; & 0 \le x < t \\ \left(1 - \frac{2x}{\pi}\right)t ; & t < x \le \pi/2 \end{cases}$

(B) $G(x,t) = \begin{cases} \left(1 - \frac{2x}{\pi}\right) ; & 0 \le x < t \\ \left(1 - \frac{2t}{\pi}\right) ; & t < x \le \pi/2 \end{cases}$

(C) $G(x,t) = \begin{cases} t\left(1 + \frac{2x}{\pi}\right) ; & 0 \le x < t \\ \left(1 + \frac{2x}{\pi}\right) ; & t < x \le \pi/2 \end{cases}$

(D) $G(x,t) = \begin{cases} x\left(1 + \frac{2t}{\pi}\right) ; & 0 \le x < t \\ \left(1 + \frac{2x}{\pi}\right) ; & t < x \le \pi/2 \end{cases}$

52. The solution u(x, y) of a Poisson's equation in a square

$u_{xx} + u_{yy} = -1, |x| \le 1, |y| \le 1, u = 0$ at $x = \pm 1$ and at $y = \pm 1$ is

(A) $\frac{5}{6}(1-x)(1-y^2)$

(B) $\frac{5}{16}(1-x^2)(1-y)$

(C) $\frac{5}{16}(1+x^2)(1+y^2)$

(D) $\frac{5}{16}(1-x^2)(1-y^2)$

53. If H_G is the iteration matrix of Gauss-Seidal Scheme, H_J is the iteration matrix of Jacobi scheme for solving AX = b, then which one of the following is true ?

(A) $\rho(H_G) = [\rho(H_J)]^2$

(B) $\rho(H_G) = \sqrt{\rho(H_J)}$

(C) $\rho(H_G) = \rho(H_J)$

(D) $\rho(H_G) = [\rho(H_J)]^3$

54. Let $\rho(H)$ be the spectral radius of the iteration matrix H, the rate of convergence of the iterative method is

(A) $\gamma = e^{\rho(H)}$

(B) $\gamma = \log_{10}[\rho(H)]$

(C) $\gamma = -\log_{10}[\rho(H)]$

(D) $\gamma = 2\log_{10}[\rho(H)]$

55. If E, Δ, ∇, δ are the shift, forward, backward and central difference operators respectively, then which of the following is not true ?

(A) $\Delta = E - 1$

(B) $\nabla = 1 - E^{-1}$

(C) $\delta = E - \Delta$

(D) $\delta = E^{1/2} - \nabla$

56. Which of the following is the value of $f'(2.0)$ obtained by using the method based on linear interpolation for the following data ?

i	0	1	2
x_i	2.0	2.2	2.6
f_i	0.69315	0.78846	0.95551

(A) 0.49619

(B) 0.19642

(C) 0.47655

(D) – 0.47655

57. If $u_{xx} + u_{yy} = 0$ in a given region, then u(x, y) cannot have a relative maximum or minimum inside the region unless

(A) u(x, y) = sinx cosy

(B) $u(x, y) = x^2\sqrt{y}$

(C) u(x, y) is constant

(D) $u(x, y) = \sqrt{x}\ y^2$

58. The integral equation for the boundary value problem $\frac{d^2y}{dx^2} - \lambda y = 0$ with y(a) = y(b) = 0 can be transformed to

(A) Non homogeneous Fredholm integral equation

(B) Homogeneous Fredholm integral equation

(C) Volterra integral equation of first kind

(D) Volterra integral equation of second kind

59. The solution of the integral equation

$\phi(x) = x + \int_0^x (\xi - x)\phi(\xi)d\xi$ is

(A) cosx

(B) sinx

(C) $\sin^{-1}x$

(D) tanx

60. In a simple pendulum of fixed length l and bob mass m, θ being the generalized co-ordinate, then for small θ, the Lagrangian of this system can be written as

(A) $L(\theta,\dot{\theta}) = \frac{1}{2}m(l^2\dot{\theta}^2 - g\,l\,\theta^2)$

(B) $L(\theta,\dot{\theta}) = \frac{1}{2}m(l^2\dot{\theta}^2 - g\,\theta)$

(C) $L(\theta,\dot{\theta}) = \frac{1}{2}m(l^2\dot{\theta} - g\,l\,\theta)$

(D) $L(\theta,\dot{\theta}) = \frac{1}{2}m(l^2\dot{\theta}^2 + g\,l\,\theta)$

61. In a Markov chain, states i and j are communicating. Then which one of the following is **not** true ?

(A) Either both are transient or both are persistent

(B) Both have same period

(C) If j is aperiodic, then stationary solution $\lim_{n\to\infty} p_{ij}^{n} = {}^{1}/_{\mu_{ij}}$, where μ_{ij} is the expected number of transitions to state j

(D) $p_{ii} = p_{jj}$

62. A test statistic is distribution-free under the null hypothesis means

(A) The distribution of the test statistic does not depend on parameter θ

(B) The distribution of the test statistic is always continuous

(C) The distribution of the test statistic does not depend on the distribution from which sample was drawn

(D) The distribution of the test statistic is a discrete distribution always

63. Data on rainfall for the month of June 2016 is available for Bengaluru city. Which one of the following tests is most appropriate when the distribution of rainfall is random ?

(A) Run test

(B) Wilcoxon Rank-Sum test

(C) Sign test

(D) Median test

64. Let $X_1, X_2, ..., X_n$ be iid Poisson distribution with mean λ. Given that $\overline{X}$ is $\subset$ AN estimator with mean λ and variance $\frac{\lambda}{n}$. Then $\subset$ AN estimator of $e^{-\lambda}$ with asymptotic mean and variance is

(A) $\bar{e}^{\overline{X}} \sim AN\left(e^{-\lambda}, \frac{e^{-2\lambda}}{n}\right)$

(B) $\bar{e}^{\overline{X}} \sim AN\left(e^{-\lambda}, e^{-2\lambda}\frac{\lambda}{n}\right)$

(C) $\bar{e}^{\overline{X}} \sim AN\left(e^{-\lambda}, e^{-\lambda}\frac{\lambda^2}{n}\right)$

(D) $\bar{e}^{\overline{X}} \sim AN\left(\lambda, \frac{\lambda^2}{n}\right)$

65. For a random sample of size n from the Poisson distribution with parameter λ, the MLE of $\Psi(\lambda) = (1 + \lambda).e^{-\lambda}$ is

(A) $\left(1+n\sum X_i\right).e^{-\overline{X}}$

(B) $\left(1+n\overline{X}\right).e^{-\sum X_i}$

(C) $\left(1+\overline{X}\right).e^{-n\overline{X}}$

(D) $\left(1+\overline{X}\right).e^{-\overline{X}}$

66. Let X ~ N (2, 1), Y ~ N (–1, 1) and I = (–2, 2). Which of the following is true ?

(A) $P(X \in I) > P(Y \in I)$

(B) $P(X \in I) = P(Y \in I)$

(C) $P(X \in I) < 2P(Y \in I)$

(D) $2P(X \in I) < P(Y \in I)$

67. Let X be a binomial (n, p) and $p(X = n) = \frac{3}{10}p(X = n-1)$, then

(A) $n = 10, p = 1/4$

(B) $n = 20, p = 1/4$

(C) $n = 20, p = 3/4$

(D) $n = 10, p = 3/4$

68. Let X have pdf $f(x) = 2x$, $0 < x < 1$. What is the pdf of the median of a random sample of size 3 from X ?

(A) $12x^3(1 - x^4), 0 < x < 1$

(B) $12x^3(1 - x^2), 0 < x < 1$

(C) $12x^2(1 - x^3), 0 < x < 1$

(D) $12x^3(1 - x^3), 0 < x < 1$

69. Let X have a exponential distribution with pdf $f(x . \lambda) = e^{-\lambda x}$, $x > 0$, $\lambda > 0$. Then the p^{th} $(0 < p < 1)$ quantile of this r.v. X is

(A) $\lambda \log(1 - p)$

(B) $-\lambda \log(1 - p)$

(C) $\frac{1}{\lambda}\log(1-p)$

(D) $-\frac{1}{\lambda}\log(1-p)$

70. Let $\{X_n\}$ be a sequence of independent random variables such that

$P(X_n = n) = \frac{1}{2n^2}, P(X_n = -n) = \frac{1}{2n^2}$

and $P(X_n = 0) = 1 - \frac{1}{n^2}$ for $n = 1, 2, 3, \ldots$.

Which of the following is true ?

(A) WLLN holds and SLLN holds

(B) WLLN does not hold but SLLN holds

(C) WLLN holds but SLLN does not hold

(D) Neither WLLN nor SLLN hold

71. Given that a Poisson process $N(t) = n$, the arrival times of n events $s_1, s_2, \ldots, s_n$ have the same distribution as that of n order statistics corresponding to n independent random observations from

(A) Gamma(n, λ)

(B) Uniform $(0, t)$

(C) Exponential (λt)

(D) Binomial $\left(n, \frac{1}{t}\right)$

72. Let $\{X_n\}$ be a Markov chain with state $\{0, 1, 2\}$ with following transition probability matrix

$$P = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

then $P(X_3 = 2 | X_1 = 0)$

(A) $\frac{3}{4}$

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{1}{8}$

73. Which of the following families of distributions is **not** complete ?

(A) $U(0, \theta), \theta > 0$

(B) $N(\theta, 1), -\infty < \theta < \infty$

(C) $N(0, \theta), 0 < \theta < \infty$

(D) $P(\lambda), \lambda > 0$

74. Suppose that –5, –8, 1, –10, 4, 5.7, 8 and 9 are independent observations from $U(-\theta, \theta), \theta > 0, \theta$ unknown. Then, the MLE of θ is

(A) 10

(B) 8

(C) 9

(D) 1

75. Let $X_1, X_2,..., X_n$ be a random sample from Cauchy distribution with location parameter μ, with Fisher information contained in one observation about μ is $\frac{1}{2}$. Then, the Rao-Score test statistic for testing $H_0 : \mu = 0$ against $H_1 : \mu \neq 0$ is

(A) $\frac{2}{n}\left(\sum_{i=1}^{n}\frac{2x_i}{1+x_i^2}\right)^2$

(B) $\frac{1}{n}\left(\sum_{i=1}^{n}\frac{2x_i}{1+x_i^2}\right)$

(C) $\frac{2}{n}\left(\sum_{i=1}^{n}\frac{x_i}{1+x_i^2}\right)$

(D) $\frac{2}{n}\left(\sum_{i=1}^{n}\frac{2x_i}{1+x_i}\right)^2$

76. Following matrix gives the distance of five objects {1, 2, 3, 4, 5}

$$D = \begin{bmatrix} 0 & & & & \\ 9 & 0 & & & \\ 3 & 7 & 0 & & \\ 6 & 5 & 9 & 0 & \\ 11 & 10 & 2 & 8 & 0 \end{bmatrix}$$

Clusters are formed using single linkage algorithm, then which of the following are in the same cluster ?

(A) {1, 2}

(B) {5, 4}

(C) {2, 4}

(D) {1, 4}

77. The following table gives the classification of number of members in two populations π_1 and π_2.

		Actual Membership	
		π_1	π_2
Predicted Membership	π_1	20	5
	π_2	3	25

Then the percent apparent error rate is

(A) 17.8%

(B) 12%

(C) 25%

(D) 15.1%

78. Given the hazard rate $r(t) = \lambda\alpha t^{\alpha-1},\ t > 0$, $\lambda > 0$, $\alpha > 1$, what is the corresponding pdf ?

(A) $\alpha\lambda t^{\alpha-1} e^{-\lambda t^{\alpha}}, t > 0$

(B) $\alpha\lambda^{\alpha-1} t^{\alpha-1} e^{-(\lambda t)^{\alpha}}, t > 0$

(C) $\alpha\lambda^{\alpha} t^{\alpha-1} e^{-\lambda t^{\alpha}}, t > 0$

(D) $\alpha\lambda^{\alpha} t^{\alpha}\ e^{-\lambda t^{\alpha}}, t > 0$

79. If $\{X_1, X_2, \ldots, X_n\}$ is a random sample from the pdf

$$f(x, \theta, \eta) = \frac{1}{\overline{|\theta}\,\eta^{\theta}} e^{-\frac{x}{\eta}}\ x^{\theta-1}, x > 0,$$

given that η is known, which of the following is sufficient for θ ?

(A) $X_1 + \ldots + X_n$

(B) $X_1 + \ldots + X_{n-1}$

(C) $X_1^2 + \ldots + X_n^2$

(D) $X_1^3 + \ldots + X_n^3$

80. Let X and Y be independent standard exponential random variables. Which of the following is correct ?

(A) XY has exponential distribution

(B) $\frac{X}{Y}$ has exponential distribution

(C) Maximum of X and Y has exponential distribution

(D) Minimum of X and Y has exponential distribution

81. If A_1 and A_2 are two events then $P(A_1 \cup A_2) = P(A_1) + P(A_2)$ when

(A) A_1 and A_2 are independent

(B) A_1 and A_2 are mutually exclusive

(C) A_1 is contained in A_2

(D) A_2 is contained in A_1

82. If $\{X_n, n \geq 0\}$ is a Markov chain on $\{1, 2, 3\}$ with transition probability matrix , $P = \begin{pmatrix} \frac{2}{5} & 0 & \frac{3}{5} \\ 0 & \frac{3}{5} & \frac{2}{5} \\ \frac{3}{5} & \frac{2}{5} & 0 \end{pmatrix}$,

what is its stationary distribution ?

(A) $\left(0, \frac{3}{5}, \frac{2}{5}\right)$

(B) $\left(\frac{1}{5}, \frac{2}{5}, \frac{2}{5}\right)$

(C) $\left(\frac{1}{3}, \frac{2}{3}, 0\right)$

(D) $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

83. In a deterministic inventory model with holding cost Rs. 0.04, set up cost Rs. 100 and demand rate 25 per unit time, what is the economic lot size ?

(A) $\frac{100}{\sqrt{2}}$

(B) 100

(C) 200

(D) $\frac{500}{\sqrt{2}}$

84. What is the average number of customers in an M/G/1 queueing system with arrival rate λ, service rate μ, variance of service time σ^2 and traffic intensity ρ ?

(A) $\frac{\lambda^2\sigma^2 + \rho^2}{2\lambda(1-\rho)} + \frac{1}{\mu}$

(B) $\frac{\lambda^2\sigma^2 + \rho^2}{2(1-\rho)}$

(C) $\frac{\lambda^2\sigma^2 + \rho^2}{2(1-\rho)} + \rho$

(D) $\frac{\lambda^2\sigma^2 + \rho^2}{2\lambda(1-\rho)}$

85. Which of the following gives the maximum number of estimable linear parametric functions in a linear model ?

(A) Trace of the design matrix

(B) Determinant of the design matrix

(C) Rank of the design matrix

(D) Permanent of the design matrix

86. With reference to a full rank Gauss-Markov model, which of the following is not true ?

(A) The design matrix has full column rank

(B) Every linear parametric function is estimable

(C) Least squares estimator of the parameter vector is not unique

(D) Least squares estimator of the parameter vector is unique

87. If X and Y are i.i.d. with characteristic function φ, what is the characteristic function of X – Y ?

(A) 1

(B) $|\varphi|^2$

(C) φ^2

(D) 0

88. In a BIBD with parameters v, b, r, k, λ, which of the following is not true ?

(A) $\frac{k}{\lambda v} I_v$ is a g-inverse of the information matrix

(B) $\frac{k}{\lambda v}\left(I_v - \frac{E_{vv}}{v}\right)$ is a g-inverse of the information matrix, where E_{vv} is a $v \times v$ matrix of 1's

(C) $\frac{\lambda v}{k} I_v$ is the information matrix

(D) $\frac{\lambda v}{k}\left(I_v - \frac{E_{vv}}{v}\right)$ is the information matrix

89. Given that X has t-distribution with degrees of freedom n and EX = 0 and $EX^2 = \infty$, what is n ?

(A) 1

(B) 2

(C) 3

(D) 5

90. You have two coins, a fair one with probability of head $1/2$ and an unfair one with probability of head $1/3$, but otherwise look identical. A coin is selected at random and tossed, falling head up. How likely is it that it is the fair one ?

(A) $1/3$

(B) $2/5$

(C) $1/2$

(D) $3/5$

91. If (X, Y) has probability mass function

$$P(X = k, Y = l) = \frac{n!}{k!\,l!\,(n-k-l)!}\,\frac{l}{4^k}\,\frac{l}{3^l}\left(\frac{5}{12}\right)^{n-k-l}$$

$$k, l = 0, 1, 2, \ldots, n,\ k + l \le n.$$

What is the conditional distribution of X given Y = 1 ?

(A) Binomial $\left(n-1, \frac{3}{8}\right)$

(B) Binomial $\left(n-1, \frac{5}{12}\right)$

(C) Binomial $\left(n-1, \frac{1}{3}\right)$

(D) Binomial $\left(n-1, \frac{1}{4}\right)$

92. In an auto correlated regression model, which one of the following estimator is BLUE ?

(A) Generalized least squares estimator

(B) Ordinary least squares estimator

(C) Ridge estimator

(D) Instrumental variable estimator

93. Given that the Durlin-Watson d-test statistic is zero, what is the first order autocorrelation ?

(A) Perfectly positive

(B) Zero

(C) Perfectly negative

(D) Non-negative

94. The value of the objective function at an optimal solution of the linear programming problem,

Minimize x + y subject to x – y = –5, x ≥ 0, y ≥ 0 will be

(A) –5

(B) 0

(C) 5

(D) 10

95. In a 2^4 factorial design, which of the interactions are confounded with the following blocks ?

Block 1 : (1) ad ac ab cd bd bc abcd

Block 2 : a d c b acd abd abc bcd

(A) AB

(B) AC

(C) ABC

(D) ABCD

96. The regression estimator $\bar{y} + b(\bar{X} - \bar{x})$ reduces to the ratio estimator whenever b equals to

(A) 0

(B) $\frac{\bar{y}}{\bar{x}}$

(C) $\bar{y}$

(D) 1

97. What is the probability that a particular unit is included in a sample of size n from a population of size N using SRSWOR ?

(A) $\frac{n-1}{N-1}$

(B) $\frac{n}{N}$

(C) $\frac{n}{N-1}$

(D) $\frac{N-n}{N-1}$

98. The relative precision in terms of intraclass correlation between units, of systematic sample mean with simple random sample mean (under usual notation) is

(A) $\frac{N-n}{N-1}\left[1+\rho(n-1)\right]$

(B) $\frac{N-n}{1+\rho(n-1)}$

(C) $\frac{N-1}{N-n}\left[1+\rho(n-1)\right]$

(D) $\frac{N-1}{1+\rho(n-1)}$

99. Boys arrive in a queue according to a Poisson process with rate α_1 and girls arrive in the same queue according to another Poisson process with rate α_2. The arrivals of boys and girls are independent. What is the probability that the first arrival in the queue is a girl ?

(A) $\frac{\alpha_1}{\alpha_1+\alpha_2}$ (B) $\frac{\alpha_2}{\alpha_1+\alpha_2}$

(C) $\frac{\alpha_1}{\alpha_2}$ (D) $\frac{\alpha_2}{\alpha_1}$

100. Which of the following is satisfied by the OLS residual vector in the regression model with n observations ?

(A) It is correlated with the regressor

(B) It is homoscedastic and autocorrelated

(C) It is heteroscedastic and auto correlated

(D) The number of independent components in it is n – 1

Space for Rough Work