Test Paper : **II**

Test Subject : **MATHEMATICAL SCIENCES**

Test Subject Code : **K-2618**

Roll No.

(Figures as per admission card)

OMR Sheet No. : _____

Name & Signature of Invigilator/s

Signature : _____

Name : _____

Time : 2 Hours

Maximum Marks : 200

Number of Pages in this Booklet : **16**

Number of Questions in this Booklet : **100**

ಅಭ್ಯರ್ಥಿಗಳಿಗೆ ಸೂಚನೆಗಳು

1. ಈ ಪುಟದ ಮೇಲ್ತುದಿಯಲ್ಲಿ ಒದಗಿಸಿದ ಸ್ಥಳದಲ್ಲಿ ನಿಮ್ಮ ರೋಲ್ ನಂಬರನ್ನು ಬರೆಯಿರಿ.
2. ಈ ಪತ್ರಿಕೆಯು ಬಹು ಆಯ್ಕೆ ವಿಧದ ನೂರು (100) ಪ್ರಶ್ನೆಗಳನ್ನು ಒಳಗೊಂಡಿದೆ.
3. ಪರೀಕ್ಷೆಯ ಪ್ರಾರಂಭದಲ್ಲಿ, ಪ್ರಶ್ನೆ ಪುಸ್ತಿಕೆಯನ್ನು ನಿಮಗೆ ನೀಡಲಾಗುವುದು. ಮೊದಲ 5 ನಿಮಿಷಗಳಲ್ಲಿ ನೀವು ಪುಸ್ತಿಕೆಯನ್ನು ತೆರೆಯಲು ಮತ್ತು ಕೆಳಗಿನಂತೆ ಕಡ್ಡಾಯವಾಗಿ ಪರೀಕ್ಷಿಸಲು ಕೋರಲಾಗಿದೆ.
   (i) ಪ್ರಶ್ನೆಪುಸ್ತಿಕೆಗೆ ಪ್ರವೇಶಾವಕಾಶ ಪಡೆಯಲು, ಈ ಹೊದಿಕೆ ಪುಟದ ಅಂಚಿನ ಮೇಲಿರುವ ಪೇಪರ್ ಸೀಲನ್ನು ಹರಿಯಿರಿ. ಸ್ಟಿಕ್ಕರ್ ಸೀಲ್ ಇಲ್ಲದ ಅಥವಾ ತೆರೆದ ಪುಸ್ತಿಕೆಯನ್ನು ಸ್ವೀಕರಿಸಬೇಡಿ.
   (ii) ಪುಸ್ತಿಕೆಯಲ್ಲಿ ಪ್ರಶ್ನೆಗಳ ಸಂಖ್ಯೆ ಮತ್ತು ಪುಟಗಳ ಸಂಖ್ಯೆಯನ್ನು ಮುಖಪುಟದ ಮೇಲೆ ಮುದ್ರಿಸಿದ ಮಾಹಿತಿಯೊಂದಿಗೆ ತಾಳೆ ನೋಡಿರಿ. ಪುಟಗಳು/ಪ್ರಶ್ನೆಗಳು ಕಾಣೆಯಾದ ಅಥವಾ ದ್ವಿಪ್ರತಿ ಅಥವಾ ಅನುಕ್ರಮವಾಗಿಲ್ಲದ ಅಥವಾ ಇತರ ಯಾವುದೇ ವ್ಯತ್ಯಾಸದ ದೋಷಪೂರಿತ ಪುಸ್ತಿಕೆಯನ್ನು ಕೂಡಲೆ 5 ನಿಮಿಷದ ಅವಧಿ ಒಳಗೆ, ಸಂವೀಕ್ಷಕರಿಂದ ಸರಿ ಇರುವ ಪುಸ್ತಿಕೆಗೆ ಬದಲಾಯಿಸಿಕೊಳ್ಳಬೇಕು. ಆ ಬಳಿಕ ಪ್ರಶ್ನೆಪತ್ರಿಕೆಯನ್ನು ಬದಲಾಯಿಸಲಾಗುವುದಿಲ್ಲ, ಯಾವುದೇ ಹೆಚ್ಚು ಸಮಯವನ್ನೂ ಕೊಡಲಾಗುವುದಿಲ್ಲ.
4. ಪ್ರತಿಯೊಂದು ಪ್ರಶ್ನೆಗೂ (A), (B), (C) ಮತ್ತು (D) ಎಂದು ಗುರುತಿಸಿದ ನಾಲ್ಕು ಪರ್ಯಾಯ ಉತ್ತರಗಳಿವೆ. ನೀವು ಪ್ರಶ್ನೆಯ ಎದುರು ಸರಿಯಾದ ಉತ್ತರದ ಮೇಲೆ, ಕೆಳಗೆ ಕಾಣಿಸಿದಂತೆ ಅಂಡಾಕೃತಿಯನ್ನು ಕಪ್ಪಾಗಿಸಬೇಕು.

   ಉದಾಹರಣೆ : (A) (B) ● (D)

   (C) ಸರಿಯಾದ ಉತ್ತರವಾಗಿದ್ದಾಗ.
5. ಈ ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯ ಜೊತೆಯಲ್ಲಿ ಕೊಟ್ಟಿರುವ **OMR ಉತ್ತರ ಹಾಳೆಯಲ್ಲಿ ನಿಮ್ಮ ಉತ್ತರಗಳನ್ನು ಸೂಚಿಸತಕ್ಕದ್ದು.** OMR ಹಾಳೆಯಲ್ಲಿ ಅಂಡಾಕೃತಿಯಲ್ಲದೆ ಬೇರೆ ಯಾವುದೇ ಸ್ಥಳದಲ್ಲಿ ಉತ್ತರವನ್ನು ಗುರುತಿಸಿದರೆ, ಅದರ ಮೌಲ್ಯಮಾಪನ ಮಾಡಲಾಗುವುದಿಲ್ಲ.
6. OMR ಉತ್ತರ ಹಾಳೆಯಲ್ಲಿ ಕೊಟ್ಟ ಸೂಚನೆಗಳನ್ನು ಜಾಗರೂಕತೆಯಿಂದ ಓದಿರಿ.
7. ಎಲ್ಲಾ ಕರಡು ಕೆಲಸವನ್ನು ಪುಸ್ತಿಕೆಯ ಕೊನೆಯಲ್ಲಿ ಮಾಡತಕ್ಕದ್ದು.
8. ನಿಮ್ಮ ಗುರುತನ್ನು ಬಹಿರಂಗಪಡಿಸಬಹುದಾದ ನಿಮ್ಮ ಹೆಸರು ಅಥವಾ ಯಾವುದೇ ಚಿಹ್ನೆಯನ್ನು, ಸಂಗತವಾದ ಸ್ಥಳ ಹೊರತು ಪಡಿಸಿ, OMR ಉತ್ತರ ಹಾಳೆಯ ಯಾವುದೇ ಭಾಗದಲ್ಲಿ ಬರೆದರೆ, ನೀವು ಅನರ್ಹತೆಗೆ ಬಾಧ್ಯರಾಗುತ್ತೀರಿ.
9. ಪರೀಕ್ಷೆಯು ಮುಗಿದನಂತರ, ಕಡ್ಡಾಯವಾಗಿ OMR ಉತ್ತರ ಹಾಳೆಯನ್ನು ಸಂವೀಕ್ಷಕರಿಗೆ ನೀವು ಹಿಂತಿರುಗಿಸಬೇಕು ಮತ್ತು ಪರೀಕ್ಷಾ ಕೊಠಡಿಯ ಹೊರಗೆ OMRನ್ನು ನಿಮ್ಮೊಂದಿಗೆ ಕೊಂಡೊಯ್ಯಕೂಡದು.
10. ಪರೀಕ್ಷೆಯ ನಂತರ, ಪರೀಕ್ಷಾ ಪ್ರಶ್ನೆಪತ್ರಿಕೆಯನ್ನು ಮತ್ತು ನಕಲು OMR ಉತ್ತರ ಹಾಳೆಯನ್ನು ನಿಮ್ಮೊಂದಿಗೆ ತೆಗೆದುಕೊಂಡು ಹೋಗಬಹುದು.
11. ನೀಲಿ/ಕಪ್ಪು ಬಾಲ್‌ಪಾಯಿಂಟ್ ಪೆನ್ ಮಾತ್ರವೇ ಉಪಯೋಗಿಸಿರಿ.
12. ಕ್ಯಾಲ್ಕುಲೇಟರ್, ವಿದ್ಯುನ್ಮಾನ ಉಪಕರಣ ಅಥವಾ ಲಾಗ್ ಟೇಬಲ್ ಇತ್ಯಾದಿಯ ಉಪಯೋಗವನ್ನು ನಿಷೇಧಿಸಲಾಗಿದೆ.
13. ಸರಿ ಅಲ್ಲದ ಉತ್ತರಗಳಿಗೆ ಋಣ ಅಂಕ ಇರುವುದಿಲ್ಲ.
14. ಕನ್ನಡ ಮತ್ತು ಇಂಗ್ಲೀಷ್ ಆವೃತ್ತಿಗಳ ಪ್ರಶ್ನೆಪತ್ರಿಕೆಗಳಲ್ಲಿ ಯಾವುದೇ ರೀತಿಯ ವ್ಯತ್ಯಾಸಗಳು ಕಂಡುಬಂದಲ್ಲಿ, ಇಂಗ್ಲೀಷ್ ಆವೃತ್ತಿಗಳಲ್ಲಿರುವುದೇ ಅಂತಿಮವೆಂದು ಪರಿಗಣಿಸಬೇಕು.

Instructions for the Candidates

1. Write your roll number in the space provided on the top of this page.
2. This paper consists of Hundred multiple-choice type of questions.
3. At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as below :
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of the cover page. Do not accept a booklet without sticker seal or open booklet.
   (ii) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.**
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.

   Example : (A) (B) ● (D)

   where (C) is the correct response.
5. Your responses to the questions are to be indicated in the **OMR Sheet kept inside this Booklet.** If you mark at any place other than in the circles in the OMR Sheet, it will not be evaluated.
6. Read the instructions given in OMR carefully.
7. Rough Work is to be done in the end of this booklet.
8. If you write your name or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
9. You have to return the test OMR Answer Sheet to the invigilators at the end of the examination compulsorily and must NOT carry it with you outside the Examination Hall.
10. You can take away question booklet and carbon copy of OMR Answer Sheet after the examination.
11. **Use only Blue/Black Ball point pen.**
12. **Use of any calculator, electronic gadgets or log table etc., is prohibited.**
13. **There is no negative marks for incorrect answers.**
14. **In case of any discrepancy found in the Kannada translation of a question booklet the question in English version shall be taken as final.**

MATHEMATICAL SCIENCES
Paper – II

Note : This paper contains **hundred (100)** objective type questions of **two (2)** marks **each**. Answer **all** questions.

1. For all $x > 0$, the inequality $e^x > x^t$ holds if and only if
   (A) $t > e^{-1}$
   (B) $t > e$
   (C) $t < e$
   (D) $t < e^{-1}$

2. Let the sequence $\{x_n\}$ be defined by $x_1 = 1$, $x_2 = 2$ and $x_{n+1} = x_n + x_{n-1}$, for $n \geq 2$. Then $\lim\limits_{n \to \infty} \dfrac{x_{n+1}}{x_n} =$

   (A) $\dfrac{\sqrt{3} - 1}{2}$

   (B) $\dfrac{1 + \sqrt{2}}{2}$

   (C) $\dfrac{1 + \sqrt{3}}{2}$

   (D) $\dfrac{1 + \sqrt{5}}{2}$

3. $\lim\limits_{n \to \infty} \prod\limits_{k=1}^{n} \cos \dfrac{\pi}{2^k} =$

   (A) $\dfrac{\pi}{2}$
   (B) $\dfrac{2}{\pi}$
   (C) 0
   (D) 1

4. $\dfrac{1}{\pi} \int\limits_{-\pi}^{\pi} x^5 \cos 5x \, dx =$

   (A) 1
   (B) π
   (C) 5π
   (D) 0

5. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be some function. Then which one of the following statement is true ?

   (A) If f is continuous at a point, then f has partial derivatives at that point
   (B) If f has partial derivatives at a point, then f is continuous at that point
   (C) If f has partial derivatives at a point, then f is differentiable at that point
   (D) If f is differentiable at a point, then f has partial derivatives at that point

6. The sum of the series
   $$\dfrac{15}{16} + \dfrac{15}{16} \cdot \dfrac{21}{24} + \dfrac{15}{16} \cdot \dfrac{21}{24} \cdot \dfrac{27}{32} + \dots \text{ is}$$
   (A) $\dfrac{64}{9}$
   (B) $\dfrac{47}{9}$
   (C) $\dfrac{56}{9}$
   (D) $\dfrac{55}{9}$

7. The sum of the series
   $$1 + \dfrac{1}{3} + \dfrac{1}{5} + \dfrac{1}{7} - \dfrac{1}{2} - \dfrac{1}{4} + \dfrac{1}{9}$$
   $$+ \dfrac{1}{11} + \dfrac{1}{13} + \dfrac{1}{15} - \dfrac{1}{6} - \dfrac{1}{8} + \dots \text{ is}$$
   (A) $\dfrac{3}{2} \log 2$
   (B) $\dfrac{5}{2} \log 2$
   (C) $\dfrac{1}{2} \log 2$
   (D) $\log \sqrt[3]{2}$

8. The series $\sum\limits_{k=1}^{\infty}\left(\dfrac{1}{k}+(-1)^k\dfrac{1}{k^2}\right)$ is

(A) Convergent

(B) Oscillating

(C) Divergent

(D) Conditionally convergent

9. Let $\{a_n\}$ be a sequence of positive real numbers such that $\lim\limits_{n\to\infty}\dfrac{a_{n+1}}{a_n}<1.$ Then

$\lim\limits_{n\to\infty}a_n =$

(A) 0

(B) 1

(C) ∞

(D) limit does not exist

10. If $a_n > 0$ for all $n \geq 1$ and $\sum\limits_{n=1}^{\infty}a_n$ converges,

then $\sum\limits_{n=1}^{\infty}\sqrt{a_n a_{n+1}}$

(A) converges

(B) diverges

(C) oscillates

(D) converges to the same sum as $\sum\limits_{n=1}^{\infty}a_n$

11. $\lim\limits_{n\to\infty}\left(\sqrt[n+1]{(n+1)!}-\sqrt[n]{n!}\right)=$

(A) e (B) e^2

(C) e^{-1} (D) e^{-2}

12. Which one of the following series diverges ?

(A) $1+\dfrac{1}{\sqrt{2}}-\dfrac{2}{\sqrt{3}}+\dfrac{1}{\sqrt{4}}+\dfrac{1}{\sqrt{5}}-\dfrac{2}{\sqrt{6}}+...$

(B) $\sum\limits_{n=1}^{\infty}(-1)^n\, n^{\frac{1-n}{n}}$

(C) $\sum\limits_{n=2}^{\infty}\dfrac{(-1)^n}{\log n}$

(D) $\sum\limits_{n=1}^{\infty}\dfrac{(-2)^n}{n^2}$

13. Consider the subspace of $\mathbb{R}^6$ spanned by the columns of the following 6×8 matrix . What is its dimension ?

$$\begin{pmatrix} 0 & -2 & 0 & -3 & 10 & 9 & 10 & 1.5 \\ 0 & 0 & 0 & -8 & 10 & 9 & 10 & 2.5 \\ 0 & 0 & 0 & 0 & 1 & 0 & 4 & 3.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3 & 5 & 4.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

(A) 6 (B) 5

(C) 4 (D) 3

14. What is the shortest distance in $\mathbb{R}^3$ between the point $(1, 2, 3)$ and the plane $x + y + z = 0$?

(A) 6

(B) $3\sqrt{2}$

(C) $2\sqrt{3}$

(D) $\sqrt{6}$

15. Which one of the following is an eigenvalue of the matrix below ?

$$\begin{pmatrix} 4 & 4 & 4 & 4 \\ 5 & 5 & 5 & 5 \\ -7 & -7 & -7 & -7 \\ 2 & 2 & 2 & 2 \end{pmatrix}$$

(A) 4   (B) 5

(C) –7   (D) 2

16. If a 3×3 real matrix A has eigenvalues 2, $\frac{3}{2}$ and $-\frac{1}{2}$, which one of the following is an eigenvalue of $8A^3 + 5I - 4A^{-2}$, where I denotes the identity matrix of size 3×3 ?

(A) -12 (B) -11

(C) -10 (D) -9

17. Suppose that M is a 4×7 real matrix that is row equivalent to the following matrix

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Which of the following statement about M is not necessarily true ?

(A) All entries in the first column of M are zero

(B) The first three rows of M are linearly independent

(C) The third and fourth column of M are linearly dependent

(D) The second, fifth and seventh columns of M span its column space

18. Consider the similarity equivalence relation on 6×6 real matrices with characteristic polynomial $(t - 6)^6$ and minimal polynomial $(t - 6)^3$. How many equivalence classes are there ?

(A) 3   (B) 4

(C) 5   (D) 6

19. Let A be 4×5 real matrix. Consider the system $A\underline{x} = \underline{b}$ of linear equations where $\underline{x}$ is a 5×1 column matrix of indeterminates and $\underline{b}$ is some fixed 4×1 column matrix with real entries. Suppose that A is row equivalent to the matrix R below and that $\underline{c}$ and $\underline{d}$ below are both solutions to $A\underline{x} = \underline{b}$. (The entry z in $\underline{d}$ is unknown at the moment)

$$R = \begin{pmatrix} 1 & -2 & -1 & -3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \underline{c} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}, \underline{d} = \begin{pmatrix} z \\ 3 \\ 4 \\ 5 \\ 5 \end{pmatrix}.$$

What is the value of z ?

(A) 4   (B) 5

(C) 6   (D) 7

20. The rank and signature of the quadratic form $-xy + z^2$ in three variables over the reals is

(A) (3, 3)   (B) (2, 1)

(C) (3, 1)   (D) (3, 2)

21. Which one of the following is false ?

(A) A skew-symmetric matrix of odd order is singular

(B) Two similar matrices have the same minimal polynomial

(C) A matrix B is nilpotent if and only if its trace is zero

(D) If a matrix A is similar to a diagonal matrix, then A is similar to its transpose

22. If A and B are square matrices of order n, which one of the following is never possible ?

(A) rank (A + B) > rank (A) + rank (B)

(B) rank (A + B) < rank (A) + rank (B)

(C) rank (A + B) = rank (A) + rank (B)

(D) rank (A + B) = rank (A) – rank (B)

23. In a group of order 15, the number of subgroups of order 3 is

(A) 3   (B) 5

(C) 1   (D) 2

24. Number of positive integers < 51 and divisible by 2 or 3 is

(A) 30   (B) 34

(C) 25   (D) 33

25. If G is an arbitrary group of even order 2n, then

(A) G has a proper normal subgroup which is not trivial

(B) G admits a quotient group of order n

(C) G has a subgroup of order 2

(D) G admits a quotient group of order 2

26. If Z is the centre of a group G of order 121, then

(A) Z is a trivial group

(B) Z ≠ G

(C) Z must be equal to G

(D) Z is always a cyclic group

27. If R is an Euclidean domain then which one of the following is true ?

(A) The polynomial ring R[X] is also an Euclidean domain

(B) The polynomial ring R[X] is a principal ideal domain

(C) Every ideal of R is principal

(D) Every quotient ring R is a domain

28. If F is a degree 4 extension field of the field of rational numbers Q, then which one of the following is true ?

(A) F is always a Galois extension of Q with Galois group cyclic

(B) F need not be a Galois extension of Q

(C) F is always a Galois extension of Q with Galois group may not be cyclic

(D) F is never a Galois extension of Q

29. For $n \geq 2$, let $(\mathbb{Z}/n\mathbb{Z})^*$ be the group of units of $\mathbb{Z}/n\mathbb{Z}$. Which one of the following is cyclic ?

(A) $(\mathbb{Z}/8\mathbb{Z})^*$ (B) $(\mathbb{Z}/15\mathbb{Z})^*$

(C) $(\mathbb{Z}/10\mathbb{Z})^*$ (D) $(\mathbb{Z}/35\mathbb{Z})^*$

30. Which one of the following cannot be the order of a finite field ?

(A) 25   (B) 21

(C) 16   (D) 9

31. The number of group homomorphisms from the group $\mathbb{Z}/5\mathbb{Z}$ onto the group $\mathbb{Z}/3\mathbb{Z}$ is

(A) 0   (B) 15

(C) 3   (D) 1

32. Let p be a prime number greater than 1. Which one of the following statement is true ?

(A) Upto isomorphism, there is a unique field of order p^2

(B) There may not be any field whose order is p^3

(C) Any field of order p^2 is contained in a field of order p^3

(D) If n > 1 is an integer, then there are non-isomorphic fields F_1 and F_2 such that $|F_1| = |F_2| = p^n$

33. Which one of the following sequences is bounded and of finite range but does not have a limit ?

(A) $\{i^n\}, i = \sqrt{-1}$ (B) $\left\{\dfrac{(-1)^n}{n}\right\}$

(C) $\left\{1 + \dfrac{(-1)^n}{n}\right\}$ (D) $\{n^2\}$

34. One of the values of i^i, $i = \sqrt{-1}$ is

(A) $e^{-\pi}$ (B) $e^{\pi/2}$

(C) $e^{-\pi/2}$ (D) e^{π}

35. Let f and g be two entire functions such that for all $z \in \mathbb{C}$, Re f (z) ≤ k Re g(z), for some real constant k. Then |f(z) – kg(z)| is

(A) $k|z^2|$

(B) ≤ k|z|

(C) a constant

(D) $k|e^z|$

36. Let $f : \mathbb{C} \to \mathbb{C}$ be a non-constant entire function. Then

(A) $f(\mathbb{C})$ is finite

(B) $\overline{f(\mathbb{C})}$ is compact

(C) $\overline{f(\mathbb{C})} = \mathbb{C}$

(D) $f(\mathbb{C})$ is not dense in $\mathbb{C}$

37. $\displaystyle\int_{|z|=1} \dfrac{z^{n-1}}{3z^n - 1} \, dz =$

(A) $\dfrac{2\pi i}{3}$

(B) $\dfrac{\pi i}{3}$

(C) $\dfrac{4\pi i}{3}$

(D) $-\dfrac{\pi i}{3}$

38. The harmonic conjugate of u = cosx coshy is

(A) v = sinx sinhy

(B) v = – cosx sinhy

(C) v = – sinx sinhy

(D) v = – sinx coshy

39. The function $f(z) = \dfrac{\pi \cot \pi z}{z^2}$ has

(A) a pole of order 3 at z = 0 and a simple pole at z = 1

(B) a pole of order 2 at z = 0 and a simple pole at z = 2

(C) a pole of order 3 at z = 0 and a pole of order 2 at z = 1

(D) a pole of order 2 at z = 0 and a pole of order 2 at z = 2

40. Suppose $x^2 + y + 4i$ and $-3 + ix^2y$ are conjugate to each other and x, y are real. Then (x, y) could be

(A) (1, 4)   (B) (1, – 4)

(C) (–1, 4)   (D) (2, 4)

41. If $z \in \mathbb{C}$, then |sin z| is

(A) bounded by 1

(B) unbounded

(C) bounded by 2π

(D) not periodic

42. The set $\{z \in \mathbb{C} : |z - 4| = |z + 4|\}$ is

(A) an ellipse in the complex plane

(B) a circle in the complex plane

(C) a point in the complex plane

(D) a line in the complex plane

43. The boundary of $A = \{x \times 0| -1 < x < 1\}$ in $\mathbb{R}^2$ is

(A) $\{-1 \times 0, 1 \times 0\}$

(B) $\{x \times 0| -1 < x < 1\}$

(C) $\{x \times 0| -1 \le x \le 1\}$

(D) $\{x \times 0| x \in \mathbb{R}\}$

44. Let $\mathbb{R}$ have the standard topology and let $f : \mathbb{R} \to A = \{a, b, c\}$ be defined by

$$f(x) = \begin{cases} a, & \text{if } x = 0 \\ b, & \text{if } x > 0 \\ c, & \text{if } x < 0 \end{cases}$$

Then the quotient topology induced by f on A is

(A) $\{\phi, A, \{a\}, \{b\}, \{b, c\}, \{a, b\}\}$

(B) $\{\phi, A, \{a\}, \{c\}, \{a, c\}\}$

(C) $\{\phi, A, \{a\}, \{b\}, \{a, b\}\}$

(D) $\{\phi, A, \{b\}, \{c\}, \{b, c\}\}$

45. Which one of the following statement is correct ?

(A) The set Q of all rationals is not connected but locally connected in $\mathbb{R}$

(B) The set $A = \left\{ x \times \dfrac{1}{x} \middle| 0 < x \le 1 \right\}$ is compact in $\mathbb{R}^2$

(C) The set $S = \left\{ x \times \sin\dfrac{1}{x} \middle| 0 < x \le 1 \right\}$ is not compact in $\mathbb{R}^2$

(D) The set $A = \left\{ \dfrac{1}{n} \middle| n \in \mathbb{Z}^+ \right\}$ is compact in $\mathbb{R}$

46. Let X be a set consisting of three elements. How many different topologies are there on X each consisting of exactly five open sets ? (These five to include the empty set and the whole set X)

(A) 3   (B) 6

(C) 12   (D) 20

47. Let $f : X \to Y$ be a continuous surjective map from a connected and compact space X to a connected and Hausdorff space Y. Which of the following is not necessarily true for f ?

(A) The image under f of a closed set of X is closed in Y

(B) The inverse image under f of a compact set of Y is compact

(C) The inverse image under f of a connected set of Y is connected

(D) The inverse image under f of a disconnected set of Y is disconnected

48. Let A be a subset of a topological space X. Then which one of the following is not correct ?

(A) A is closed if and only if it contains all its limit points

(B) A is closed if and only if $BdA \subset A$

(C) A is open if and only if $A \cap BdA = \phi$

(D) $X \neq Int\ A \cup BdA \cup Int(A^c)$

49. If $\phi(x)$ is a solution of the differential equation $\dfrac{dy}{dx} + (\cos x)y = \sin x \cos x$

then $\phi(x)$ is equal to

(A) $(\sin x - 1) + c\ e^{-\sin x}$, where c is an arbitrary constant

(B) $(\sin x - 1) + c\ e^{\sin x}$, where c is an arbitrary constant

(C) $(\sin x + 1) + e^{\cos x}$

(D) $(\sin x + 1) + 2e^{\sin x}$

50. The roots m_1, m_2 of the auxiliary equation for the system of differential equations

$\dfrac{dx}{dt} = x + y;$

$\dfrac{dy}{dt} = 4x - 2y$, are

(A) $(3, -2)$

(B) $(-3, -2)$

(C) $(-3, 2)$

(D) $(3, 2)$

51. The critical point $(0, 0)$ of the linear system

$\dfrac{dx}{dt} = a_1 x + b_1 y;$

$\dfrac{dy}{dt} = a_2 x + b_2 y,$

is stable if and only if both roots of the auxiliary equation of the system have

(A) Positive real parts

(B) Equal real parts

(C) Non-positive real parts

(D) Non-equal positive real parts

52. The general second order partial differential equation (p.d.e.) $Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = G$ is called

(A) Elliptic p.d.e. if $B^2 - 4AC = 0$

(B) Elliptic p.d.e. if $B^2 - 4AC > 0$

(C) Elliptic p.d.e. if $B^2 - 4AC < 0$

(D) Hyperbolic p.d.e. if $B^2 - 4AC < 0$

53. The integral surface satisfying the linear partial differential equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ containing the straight line $x + y = 0$, $z = 1$, is

(A) $x^2 - y^2 + 2xyz - 2z + 2 = 0$

(B) $x^2 + y^2 + 2xyz - 2z + 2 = 0$

(C) $x^2 + y^2 - 2xyz - 2z + 2 = 0$

(D) $x^2 + y^2 + 2xyz + 2z + 2 = 0$

54. The function

$$u_n(x,t) = \left(A_n \cos \frac{n\pi ct}{L} + B_n \sin \frac{n\pi ct}{L} \right) \sin \frac{n\pi x}{L}$$

is the solution of

(A) One dimensional wave equation when the string is clamped in position at $x = 0$ and $x = L$

(B) One dimensional heat equation with $u(0, t) = 0$, $u(L, t) = 0$

(C) Poisson's equation with $u(x, 0) = u(L, t) = 0$

(D) Two dimensional heat equation

55. An iterative formula by Newton-Raphson's method to compute the reciprocal of a natural number N, is

(A) $x_{n+1} = x_n(2 - Nx_n)$

(B) $x_{n+1} = x_n(1 - Nx_n)$

(C) $x_{n+1} = x_n^2(x_n - N)$

(D) $x_{n+1} = x_n(2 + Nx_n)$

56. For the evaluation of $\int_a^b f(x)dx$, the Simpson's one-third rule requires the interval [a, b] to be divided into

(A) an odd number of subintervals of equal width

(B) an even number of subintervals of equal width

(C) any number of subintervals of non equal width

(D) an even number of subintervals of variable width

57. The necessary and sufficient condition for convergence of an iterative method of the form $X^{(K+1)} = HX^{(K)} + C$, $K = 0, 1, 2,...$ is that, the eigenvalues λ_i of the iteration matrix H satisfy

(A) $|\lambda_i(H)| = 1$

(B) $|\lambda_i(H)| > 1$

(C) $|\lambda_i(H)| < 1$

(D) $|\lambda_i(H)| \geq 1$

58. The extremal of the functional

$\int_0^1 (1 + y''^2)dx$ with $y(0) = 0$, $y'(0) = 1$,

$y(1) = 1$, $y'(1) = 1$, is

(A) $y = x^3$

(B) $y = x^2$

(C) $y = x$

(D) $y = \sqrt{x}$

59. The shortest distance between the parabola $y = x^2$ and the straight line $x - y = 5$ is

(A) $\frac{19}{8}\sqrt{2}$

(B) $\frac{19}{7}\sqrt{2}$

(C) $\frac{19}{8}\sqrt{3}$

(D) $\frac{8}{19}\sqrt{2}$

60. If a body is moving under no external forces about a fixed point in the body, then, which one of the following is true ?

(A) The kinetic energy of the system is not constant

(B) The angular momentum is unaltered during motion

(C) The potential energy of the system is not constant

(D) The angular momentum varies during the motion

61. The expectation of the total number of points obtained when three fair dice are rolled together once is

(A) 10.5   (B) 6.0

(C) 12.5   (D) 11.0

62. Let $\{X_n\}$ be a sequence of independent random variables with

$$P[X_n = 0] = 1 - \frac{1}{n}, P[X_n = 1] = \frac{1}{n}.$$

n = 1, 2,... Then

(A) $X_n \to 0$ almost surely

(B) $X_n \to 1$ in probability

(C) $X_n \to 0$ in mean square

(D) $X_n \to 1$ in mean square

63. Which of the following statement is not true ?

(A) Difference of two independent Poisson processes is again a Poisson process

(B) Homogeneous Poisson process is Markovian

(C) Inter-arrival time distribution in a Poisson process is exponential

(D) Poisson process is a renewal process

64. In a regression model with non-zero mean error, which of the following is true for the OLS estimators of the regression coefficient β and error variance σ^2 ?

(A) Both are unbiased and consistent

(B) Both cannot be simultaneously unbiased and consistent

(C) Both are consistent

(D) Both are unbiased

65. Which of the following functions defined on $(-\infty, \infty)$ is not a characteristic function ?

(A) 1

(B) e^{it}

(C) e^{-it}

(D) $e^{it} + e^{-it}$

66. If the mean of a Poisson random variable X is 5 then $E(X^2) =$

(A) 25

(B) $\frac{1}{5}$

(C) 30

(D) $\sqrt{5}$

67. If X follows F distribution with degrees of freedom m and n then $\frac{1}{X}$ follows

(A) t-distribution with m + n degrees of freedom

(B) Chi-square distribution with m + n degrees of freedom

(C) Exponential distribution with mean m + n

(D) F-distribution with degrees of freedom n and m

68. Which of the following is always the lowest significance level at which a null hypothesis is rejected ?

(A) t-value

(B) significance level

(C) confidence level

(D) p-value

69. A test function is said to be unbiased if

(A) its power is less than the size

(B) its power is greater or equal to the size

(C) its significance level is pre-specified

(D) the test function is estimable

70. If $a'\theta$ is an estimable linear parametric function in a less than full rank Gauss-Markov model $(Y, A\theta, \sigma^2 I_n)$, then which one of the following is the BLUE of $a'\theta$?

(A) $a'(A'A)^{-1}A'y$, where y is a realization of Y

(B) $a'(A'A)^-A'Ay$, where y is a realization of Y

(C) $a'(A'A)^{-1}(A'A)^- y$, where y is a realization of Y

(D) $a'(A'A)^- A'y$, where y is a realization of Y

71. Let Y_1, Y_2 be independent random variables following the linear model, $Y_1 = \alpha + \varepsilon_1$, $Y_2 = 2\alpha + \varepsilon_2$. Which of the following is the least squares estimator of α ?

(A) $\dfrac{1}{5}(2Y_1 + Y_2)$

(B) $\dfrac{1}{5}(Y_1 + 2Y_2)$

(C) $2Y_1 + Y_2$

(D) $Y_1 + 2Y_2$

72. Let $Y \sim N_p(\mu, \sigma^2 I_p)$ and A, B be two conformable matrices. Then the random vectors AY and BY are independent if and only if

(A) $AB = 0$

(B) $AB = I$

(C) $(AB)^2 = AB$

(D) $AB = BA \neq I$

73. If a 3×1 vector Y is distributed as $N_3(0, I_3)$ and $B = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$, then the distribution of Y'BY is

(A) $N_3(0, I_3)$

(B) Chi-square with 1 degree of freedom

(C) Chi-square with 2 degrees of freedom

(D) Chi-square with 3 degrees of freedom

74. If ρ_{wsy} denotes the intra-class correlation coefficient in a systematic sampling, then the variance of the estimator under systematic sample is less than the variance of the estimator under simple random sampling with replacement if

(A) $\rho_{wsy} > 0$

(B) $\rho_{wsy} > \dfrac{1}{n-1}$

(C) $\rho_{wsy} \le 0$

(D) $\rho_{wsy} \le -\dfrac{1}{n-1}$

75. RBD is an example of which one of the following ?

(A) Disconnected design

(B) Connected, balanced and orthogonal design

(C) Connected and balanced but not orthogonal design

(D) Connected and orthogonal but not balanced design

76. Given the block A, B , AC, BC, which of the following is confounded ?

(A) B

(B) AB

(C) AC

(D) ABC

77. Three components with failure rates 25×10^{-3}, 32×10^{-3} and 43×10^{-3} are arranged in series. What is the system failure rate ?

(A) 25×10^{-3}

(B) 43×10^{-3}

(C) 10^{-1}

(D) 1

78. In an M/M/1/∞/FIFO queue with arrival rate 3 and service rate 4, the mean queue length under steady state is

(A) $\dfrac{4}{3}$

(B) $\dfrac{3}{4}$

(C) 3

(D) $\dfrac{9}{4}$

79. The feasible region of a linear programming problem is

(A) Convex

(B) Concave

(C) Spherical

(D) Cylindrical

80. If X and Y are independent Poisson (1) random variables, what is the distribution of X + Y ?

(A) Poisson (2)

(B) Poisson (1)

(C) Binomial (2, ½)

(D) Geometric (½)

81. If X and Y are independent Poisson (2) random variables, what is the conditional expectation of X given X + Y = 10 ?

(A) 10

(B) 2

(C) 1

(D) 5

82. If X_1, X_2,..., X_n are i.i.d. standard exponential, what is the distribution of $n \times \min \{X_1,..., X_n\}$?

(A) Exponential with mean n

(B) Exponential with mean 1

(C) Exponential with mean $\dfrac{1}{n}$

(D) Exponential with mean –1

83. If $(X_1, ..., X_d)$ is a d-variate normal random vector, which one of the following need not be true ?

(A) All marginals are normal

(B) All linear combinations are normal

(C) Variance-covariance matrix is positive definite

(D) Conditional distribution of X_1 given X_2,..., X_d is normal

84. If X and Y are i.i.d. exponential with mean 1, what is the distribution of $\dfrac{X}{X+Y}$?

(A) F with degrees of freedom 1, 2

(B) Uniform over (0, 1)

(C) Standard Cauchy

(D) Normal (½, 2)

85. Given that F, G, H are distribution functions and $F_1 = \dfrac{F+G+H}{3}$, $F_2 = F^2 GH$,

$F_3 = \dfrac{2F+3G+H}{6}$, $F_4 = \dfrac{F+G+H}{2}$, which of the following is true ?

(A) F_1 is a distribution function but not F_2, F_3, F_4

(B) F_2 is a distribution function but not F_1, F_3, F_4

(C) F_1 and F_2 are distribution functions but not F_3 and F_4

(D) F_1, F_2, F_3 are distribution functions but not F_4

86. Let (X, Y) have distribution function $F(x, y)$, $-\infty < x, y < \infty$. Which of the following is not true ?

(A) For a fixed y_0, $F(x, y_0)$ is non-decreasing and right continuous

(B) For any x, $F(x, \infty) = 1$

(C) For any x, $F(-\infty, x) = 0$

(D) For $y_1 < y_2$, $F(x, y_2) - F(x, y_1) \geq 0$, $-\infty < x < \infty$

87. Let $\{X_n, n \geq 0\}$ be a Markov chain with states 0, 1, 2 and $p_{00} = p_{02} = p_{11} = p_{12} = p_{20} = p_{21} = ½$. What is the stationary distribution ?

(A) $\left(\dfrac{1}{6}, \dfrac{1}{6}, \dfrac{2}{3} \right)$

(B) $\left(\dfrac{1}{3}, \dfrac{1}{3}, \dfrac{1}{3} \right)$

(C) $\left(\dfrac{1}{4}, \dfrac{1}{4}, \dfrac{1}{2} \right)$

(D) $\left(\dfrac{1}{8}, \dfrac{1}{4}, \dfrac{5}{8} \right)$

88. Consider a Markov chain with states 1, 2 and $p_{12} = \dfrac{1}{3} = 1 - p_{21}$. What is $\lim_{n \to \infty} p_{11}^{(n)}$?

(A) 0

(B) 1

(C) $\dfrac{1}{3}$

(D) $\dfrac{2}{3}$

89. With reference to characteristic functions, which of the following is not true ?

(A) A characteristic function can be complex valued

(B) Product of two or more characteristic functions is a characteristic function

(C) Modulus of a characteristic function is a characteristic function

(D) A characteristic function is always real valued

90. What is a sufficient condition for a block design with 5 treatments and 5 blocks to be connected ?

(A) Rank of its design matrix is 9

(B) Rank of its design matrix is 8

(C) Rank of its design matrix is 7

(D) Rank of its design matrix is 10

91. What is the steady state distribution of the number of customs in a G/M/1 system at the arrival time points ?

(A) Uniform

(B) Binomial

(C) Poisson

(D) Geometric

92. If $Z = (U, V, W)$ and $V(Z) = \begin{pmatrix} 5 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 3 \end{pmatrix}$, what is the variance of $U - 2V + W$?

(A) 0

(B) 18

(C) 11

(D) 6

93. Let $\{X_n, n \geq 1\}$ be a sequence of independent random variables with $P(X_n = 0) = \dfrac{1}{n} = 1 - P(X_n = 1)$, $n \geq 1$. What is $P(X_n = 0$ infinitely often$)$ equal to ?

(A) 0

(B) ½

(C) 1

(D) < 1

94. If X is a random variable with unit mean then the correct inequality is

(A) $E(e^{-X}) \geq e$

(B) $E(e^{-X}) \leq e$

(C) $E(e^{-X}) \geq \dfrac{1}{e}$

(D) $E(e^{-X}) \leq \dfrac{1}{e}$

95. What is Durbin-Watson test used for in regression analysis ?

(A) Testing multicollinearity

(B) Testing the presence of autocorrelation

(C) Testing homoscedasticity

(D) Testing the relevance of regression

96. Which of the following is the correct inequality for the variances under random sampling (ran) proportional allocation (pa) and optimum allocation (opt) ?

(A) $V_{ran}(\bar{y}_{st}) \leq V_{pa}(\bar{y}_{st}) \leq V_{opt}(\bar{y}_{st})$

(B) $V_{ran}(\bar{y}_{st}) \geq V_{pa}(\bar{y}_{st}) \geq V_{opt}(\bar{y}_{st})$

(C) $V_{ran}(\bar{y}_{st}) \geq V_{opt}(\bar{y}_{st}) \geq V_{pa}(\bar{y}_{st})$

(D) $V_{ran}(\bar{y}_{st}) \leq V_{opt}(\bar{y}_{st}) \leq V_{pa}(\bar{y}_{st})$

97. In case of autocorrelated regression model with lagged dependent variable, the appropriate method of estimation is

(A) Generalised least squares

(B) Ridge estimator

(C) Feasible generalised least squares

(D) Instrumental variable method

98. The over all significance of the regression model is tested using

(A) ANOVA

(B) t-test

(C) R^2- coefficient of variation

(D) Adjusted-R^2

99. In a BIBD with parameters v, b, r, k, λ, which of the following is not true ?

(A) $\dfrac{r}{b} = \dfrac{k}{v}$

(B) $\dfrac{\lambda}{k-1} = \dfrac{r}{v-1}$

(C) $v \geq b$

(D) $b \geq v$

100. In a Gauss-Markov model, what is the sum of dimension of the estimation space and dimension of the error space equal to ?

(A) Number of parameters in the model

(B) Rank of the model

(C) Number of parameters minus the rank of the model

(D) Rank of the model minus the number of parameters in the model

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Space for Rough Work

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