

Registration Number:

Date & Session:



ST. JOSEPH'S UNIVERSITY, BENGALURU-27
M.Sc (BIG DATA ANALYTICS) – I SEMESTER
SEMESTER EXAMINATION: OCTOBER 2022
(Examination conducted in December 2022)
BDA1221: PROBABILITY AND STOCHASTIC PROCESS

Time: 2 Hours

Max Marks: 50

This paper contains TWO printed pages and THREE parts

PART A

Answer ALL questions

5 X 1 = 5

1. If X is a random variable, what is $\text{Cov}(X, X)$?
2. If X and Y are independent random variables, what is $\text{Cov}(X, Y)$?
3. The Uniform Distribution is continuous - True or false?
4. If X is a continuous random variable, $P(X=x) = 0$ - True or false?
5. If X has a standard normal distribution, what is the distribution of X^2 ?

PART B

Answer any FIVE questions

5 X 3 = 15

6. State the Central Limit Theorem (CLT). How do you use CLT in testing of hypotheses?
7. A, B and C are playing ball. A is twice as likely to throw the ball to B than to C. B always throws the ball to C. C is equally likely to throw to A or B. Write down the probability transition matrix for this ball game.
8. Distinguish between errors of the first and second kind. What is the 'level' and 'power' of a test?
9. Old man Ramaiah is likely to live for five more years with a probability of $3/4$. His wife Sitamma, who has cancer, only has a $1/4$ chance of surviving these five years. What's the chance that both will be dead after five years? What's the chance that at least one of them will be dead after five years?
10. What are the parameters of the Binomial theorem? If X is the outcome of 20 independent trials with a probability 0.4 of success, what is its mean and standard deviation?
11. Write down (without proof) the expansion of $P(A_1 \cup A_2 \cup A_3 \cup A_4)$.
12. When does a time series become stationary?

PART C

Answer any THREE questions.

3 X 10 = 30

13. A machine can produce chalk pieces of length 3 inches in large numbers. It is now suspected that the machine is malfunctioning, and might actually be producing chalk pieces *less than* 3 inches in length. Explain all the steps involved in testing if this suspicion is valid. Your answer must mention (a) the null and alternate hypotheses, (b) the sampling process, (c) the underlying probability distribution, (d) the testing criterion, and (e) an explanation of the p-value. [2+1+2+2+3]
14. Imagine that you have weekly data for the variation in the price of crude oil over the past 12 months. How would you analyse this data, and predict the likely crude oil price over the next three months. Your answer must include (a) a sketch of the time series plot, a discussion on (b) the likely seasonal effect, (c) an identification of any inflationary trend, and a (d) brief explanation of how you might implement the ARIMA model. [3+1+1+5]
15. Let $\{A_1, A_2 \dots A_n\}$ be n events that are "mutually exclusive" and "collectively exhaustive". (a) Explain these two concepts, and then (b) prove the Bayes theorem. (c) The probability of acquiring a deadly disease is 0.01. The probability of testing positive given that a patient has the disease is 0.99. The probability of testing negative when the patient does not have the disease is also 0.99. A random patient is found to have tested positive. What is the probability that he really has the disease? [2+5+3].
16. What is (a) the Markovian property? Explain (b) what we mean by the 'states' of a Markov chain. (c) Sketch an example of a 3-state Markov chain and write down its 3 x 3 one-step probability transition matrix (PTM). (d) How can we compute the two-step PTM, given the one-step PTM? Write down the two-step PTM for your chosen example. Finally (e) explain the idea of an 'absorbing' state and a 'reflecting' state. [2+1+3+2+2]