Register Number:

Date:

ST. JOSEPH'S UNIVERSITY, BENGALURU- 27
M.Sc MATHEMATICS- I SEMESTER
SEMESTER EXAMINATION: OCTOBER 2022
(Examination conducted in December 2022)
MT 7421- Ordinary Differential Equations

Duration: 2 Hours **Max. Marks:** 50

1. The paper contains **TWO** printed pages.
2. Answer any **FIVE FULL** questions, where each question carries 10 marks.

1. (a) If $P(D)$ and $Q(D)$ are two polynomial operators, prove that $Q(D)[P(D)u] = [Q(D)P(D)]u$.
   (b) Solve $(D^2 + 2D + 1)y = x - e^x$ using the method of undetermined coefficients. **[5+5]**
2. (a) Show that the equation $y'' - 4y' = 0$ forms the fundamental set on the interval $(-\infty, \infty)$ and write the general solution.
   (b) A tank initially contains 50 gallons of pure water. A salt solution containing 2 pounds of salt per gallon of water is poured into the tank at the rate of 3 gallons per minute. The mixture is stirred and is drained out of the tank at the same rate.
   i. Find the initial value problem that describes the amount of salt in the tank at any time.
   ii. Find the amount of salt in the tank at any time.
   iii. Find the amount of salt in the tank after 20 minutes.
   iv. Find the amount of salt in the tank after a long time. **[3+7]**
3. Find the power series solution in powers of $(x - 2)$ of the differential equation $y'' + (x - 3)y' + y = 0$.
4. Solve using Frobenius method the given differential equation $9x(1 - x)y'' - 12y' + 4y = 0$.
5. Find the eigenvalue and eigen function of the differential equation $\frac{d}{dx}\left(x\frac{dy}{dx}\right) + \frac{\lambda}{x}y = 0$ with boundary conditions $y(1) = 0$ and $y'(e^{2\pi}) = 0$.
6. State and prove Green's Identity.

7. (a) Define the critical point for an autonomous system of differential equations. Find the critical points of $\frac{d^2x}{dt^2} + \frac{c}{m}\frac{dx}{dt} + \frac{q}{a}sinx = 0$

(b) Determine the type and stability of the critical point of $(0, 0)$ of the non linear system of equation $\frac{dx}{dt} = 8x - y^2$, $\frac{dy}{dt} = -6y + 6x^2$. **[5+5]**

OR

(a) Show that $\frac{d}{dx}\{x^n J_n(x)\} = x^n J_{n-1}(x)$ where $J_n(x)$ is Bessel's function.

(b) Find the adjoint differential equation of $L_3 y = 0$, where $L_3 = \sum_{r=0}^{3} a_r(x)\frac{d^{3-r}}{dx^{3-r}}$ **[5+5]**

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