Registration Number:

Date & Session

**ST. JOSEPH'S UNIVERSITY, BENGALURU-27**
**M.Sc (MATHEMATICS) - I Semester**
**SEMESTER EXAMINATION : OCTOBER 2022**
**(Examination conducted in December 2022)**
**MT7521: DISCRETE MATHEMATICS AND GRAPH THEORY**

**Duration:** 2 Hours **Max. Marks:** 50

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1. This paper contains two pages.
2. Attempt any **FIVE FULL** questions.

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1. a) There are 2504 computer science students at a school. Of these, 1876 have taken a course in Java, 999 have taken a course in Linux and 345 have taken a course in C. Further, 876 have taken courses in both Java and Linux, 231 have taken courses in both Linux and C, and 290 have taken courses in both Java and C. If 189 of these students have taken courses in Linux, Java, and C, how many of these 2504 students have not taken a course in any of these three programming languages? **[3m]**

   b) How many positive integers less than 1000 are divisible by 7 but not by 11? **[3m]**

   c) Find the number of positive integers not exceeding 100 that are either odd or the square of an integer. **[2m]**

   d) How many different three-letter words can be formed which begin with $A$? **[2m]**

2. a) Explain the tower of Hanoi puzzle and find a recurrence relation for determining the number of moves to solve the Tower of Hanoi puzzle with $n$ disks. **[5m]**

   b) Draw the Hasse diagram for the partial ordering $\{(A, B)|A \subseteq B\}$ on the power set $P(S)$ where $S = \{a, b, c\}$. Also, determine minimal, maximal, greatest and least elements of the poset. **[5 m]**

3. Prove that a nontrivial connected digraph $D$ is Eulerian if and only if $od(v) = id(v)$ for every vertex $v$ in $D$. **[10m]**

4. If $G$ is a graph of order $n \geq 3$ and $deg(u) + deg(v) \geq n$ for each pair $u, v$ of nonadjacent vertices of $G$, then prove that $G$ is Hamiltonian. **[10m]**

5. a) Prove that every tree of order $n$ has size $n - 1$. **[4m]**

   b) State Matrix Tree Theorem. Using Matrix Tree Theorem, determine the number of spanning trees of the complete graph $K_4$. **[6m]**

6. a) For any nontrivial connected graph $G$, prove that $\alpha_0(G) + \beta_0(G) = n$ where $\alpha_0(G)$ is the vertex covering number and $\beta_0(G)$ is the vertex independence number of $G$. **[5m]**

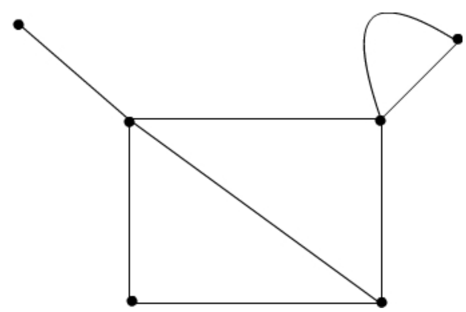
b) Prove that a graph $G$ of order $n$ is a tree if and only if its chromatic polynomial is $t(t-1)^{n-1}$. **[5m]**

7. a) Define a minimum dominating set and a minimal dominating set of a graph. Give one example each. What is the domination number of $K_9$? **[5m]**

   b) If $G$ is a graph of order $n$, then prove that $\frac{n}{1+\Delta(G)} \leq \gamma(G) \leq n - \Delta(G)$ where $\gamma(G)$ is the domination number of $G$ and $\Delta(G)$ is the maximum degree in $G$. **[5m]**

**OR**

   a) Define a cut set in a graph. For the graph shown below, find the cut set matrix. **[4m]**



   b) Prove that for every graph $G$, $\kappa(G) \leq \lambda(G) \leq \delta(G)$ where $\kappa(G)$ is the vertex connectivity, $\lambda(G)$ is the edge connectivity and $\delta(G)$ is the minimum degree of $G$. **[6m]**