Registration Number:

Date & session:

ST. JOSEPH'S UNIVERSITY, BENGALURU -27
B.Sc. (MATHEMATICS) – I SEMESTER
SEMESTER EXAMINATION: OCTOBER 2022
(Examination conducted in December 2022)
MT 121 – MATHEMATICS I

Time: 2 Hours **Max Marks: 50**

This paper contains 2 printed pages and 5 parts

I. Answer any five of the following: (5X2=10)

1. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 3 & 2 \end{bmatrix}$
2. Find the eigen values of $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$
3. Evaluate $D^n(e^x \sinh x \ cosx)$
4. If $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$ then show that $(1 + x^2)y_2 + 2xy_1 = 0$
5. Find the constant c of Rolle's theorem for the function $f(x) = x^2(1 - x)^2$ in $(0,1)$
6. Evaluate $\lim_{x\to 1}\left(\frac{x}{x-1} - \frac{1}{logx}\right)$
7. If $u = x^3 - 3xy^2 + x + e^x cosy + 1$ show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
8. If $x = rcos\theta, y = rsin\theta, z = z$ evaluate Jacobian of $x, y, z \ with\ respect\ to\ r, \theta, z.$

II. Answer any two of the following: (2X5=10)

9. Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$ by reducing to normal form.

10. Test the consistency and solve the system of equations.
$x + y + z = 6, 3x + y + z = 8, x - y + 2z = 8$

9. State Cayley-Hamilton theorem. Find the inverse of the matrix $A = \begin{bmatrix} 1 & -2 \\ 5 & -4 \end{bmatrix}$ by using Cayley-Hamilton theorem.

III. Answer any two of the following: (2X5=10)

10. Find the nth derivative of (a) $\frac{x^2}{(x+1)^2(x+2)}$ (b) $sin4xsinx$ (3+2)
11. Derive the nth derivative of $y = e^{ax}\sin(bx+c)$ hence evaluate the nth derivative of $y = e^x sin^2 x$.
12. If $y = \sin(m\sin^{-1}x)$ then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2-m^2)y_n = 0$

IV. Answer any two of the following: (2X5=10)

13. State and prove Lagrange's mean value theorem.
14. Verify Cauchy's mean value theorem for the functions $\frac{1}{x^2}$ and $\frac{1}{x}$ in the interval (a,b).
15. Expand e^{sinx} using Maclaurin's theorem up to the term containing x^4.

V. Answer any two of the following: (2X5=10)

16. If $u = f(r)$ and $x = rcos\,\theta, y = rsin\theta$ show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$
17. State Euler's theorem for homogeneous functions and If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.
18. Expand $e^x siny$ in Taylor's series around the origin up to 4 terms.
