



Register Number:

Date:

ST. JOSEPH'S UNIVERSITY, BENGALURU-27
B.Sc. (MATHEMATICS) - I SEMESTER
SEMESTER EXAMINATION: OCTOBER 2022
(Examination conducted in December 2022)
MTOE 2: MATHEMATICS FOR PHYSICS AND CHEMISTRY

Time: 2 Hours

Max Marks: 50

This question paper contains **TWO** printed pages and **FOUR** parts. Normal **calculator** is allowed to use.

I. ANSWER ANY SEVEN OF THE FOLLOWING.

(7×2=14)

1. Define a non-singular matrix and write the rank of any non-singular matrix of order n .

2. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

3. Check if the following system of linear equations is consistent or not:

$$\begin{aligned}x + y - 2z &= 5 \\x - 2y + z &= -2 \\-2x + y + z &= 4.\end{aligned}$$

4. Is every continuous function at a given point differentiable at that point? Justify your answer.

5. State Cauchy's Mean Value Theorem.

6. Evaluate $\lim_{x \rightarrow 2} \sqrt{4x^2 - 3}$.

7. Find the derivative of the function $x^3 \sin^4 x$.

8. Evaluate $\int_0^{\frac{\pi}{2}} \sin^7 x \, dx$.

9. Evaluate $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^6 x \, dx$.

II. ANSWER ANY TWO OF THE FOLLOWING.

(2×6=12)

10. Find the value of 'a' for which the following matrix has rank 3:

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ a & 2 & 2 & 2 \\ 9 & 9 & a & 3 \end{bmatrix}.$$

11. Solve the following system of linear equations:

$$\begin{aligned}x + 2y + 3z &= 0 \\y + 5z &= 0 \\3x + 2y + z &= 0 \\2x + 3z &= 0.\end{aligned}$$

12. Find the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{bmatrix} 3 & 4 \\ -2 & -3 \end{bmatrix}$.

III. ANSWER ANY TWO OF THE FOLLOWING.

(2×6=12)

13. Is $f(x) = x^2 - 2x + 3$ continuous at $x = 3$? Justify your answer using the definition of continuity.
14. Obtain the Maclaurin's series expansion of cosine function.
15. Evaluate the following limits using L'Hospital's Rule:

i $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$,

ii $\lim_{y \rightarrow \infty} \left(1 + \frac{1}{y}\right)^y$. (2+4)

IV. ANSWER ANY TWO OF THE FOLLOWING.

(2×6=12)

16. Find the arc length of the curve $y = \frac{e^x + e^{-x}}{2}$, where $0 \leq x \leq 2$.
17. Find the area bounded by the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.
18. Find the area of the surface generated by revolving the curve $x = y^3$ between $y = 0$ and $y = 2$ around the y-axis.
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