



ST. JOSEPH'S UNIVERSITY, BENGALURU -27
M.Sc Physics – I SEMESTER
SEMESTER EXAMINATION: OCTOBER 2022
(Examination conducted in December 2022)
PH7221 – Mathematical Physics

Registration Number:

Date & session:

Time: 2 Hours

Max Marks: 50

This paper contains TWO printed pages and TWO parts

PART-A

Answer any FIVE questions. Each question carries SEVEN Marks.

[5 x 7 = 35]

1. Prove Cauchy's Theorem $\oint f(z)dz = 0$. What are the conditions $f(z)$ should satisfy for this theorem to be valid? [7]
2. With the correct reasons explain whether the Fourier Transforms of a). $\sin(x)$, b). $\delta(x)$ and c). $\sin\left(\frac{1}{x}\right)$ exist or not. [7]
3. (a). Find the Fourier transform of a Gaussian function.
(b). What will be the nature of the Fourier Transform of a function that has an even real part and an odd imaginary part? What are such functions called as? [5+2]
4. (a). Prove that $\lim_{z \rightarrow 0} \frac{\bar{z}}{z}$ does not exist.
(b). Obtain the Forward transform (Jacobian Matrix) in polar and cartesian form. [3+4]
5. (a). Using Bessel's Function, show that

$$(i). J_{1/2}(x) = \sqrt{\left(\frac{2}{\pi x}\right)} \sin x$$

(b). With the help of Rodrigue's formula, find the Legendre polynomials for the given conditions,

$$(i). \int_{-1}^{+1} P_0(x). dx, (ii). \int_{-1}^{+1} \|P_0(x)\|^2. dx$$

[3+4]

6. Compute the solution of the two-dimensional Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ for the following cases (where k is separation constant) (i) $k = 0$, (ii). $k > 0$, (iii). $k < 0$. [7]
7. (a). Obtain an expression for one dimensional(1D) wave equation.
(b). Demonstrate that the Hermite polynomials $H_2(x)$ and $H_3(x)$ are orthogonal. [4+3]

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PART-A

Answer any THREE questions. Each question carries FIVE Marks.

[3 x 5 = 15]

8. Find the residues of $f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$. [5]

9. Find the Fourier transform of the derivative of a top hat function. [5]

10. Using the method of separation of variables, find the solution of the following equation,

$$\frac{\partial u}{\partial x} + u = \frac{\partial u}{\partial t} \text{ if } u = 4e^{-3x}, \text{ when } t = 0. \quad [5]$$

11. Using generating function, find the Legendre polynomials

$$(i). P_n(1), (ii). P_n(-1), (iii). P_n(-x)$$

[1+2+2]