



Register Number:
Date & Time:

ST. JOSEPH'S UNIVERSITY, BENGALURU-27
M.Sc (MATHEMATICS) - II SEMESTER
SEMESTER EXAMINATION: APRIL 2023
(Examination conducted in May 2023)
MT-8321: COMPLEX ANALYSIS
(For current batch students only)

Duration: 2 Hours

Max. Marks: 50

This paper contains **ONE** printed page and **ONE** part.

Answer any **FIVE** of the following.

1. State and Prove Rectangle Theorem I. [10]
2. (a) Show that there are no analytic functions $f = u + iv$ with $u(x, y) = x^2 + y^2$.
(b) Suppose f is an entire function of the form $f(x, y) = u(x, y) + iv(x, y)$. Show that f is a linear polynomial. [3+7]
3. Evaluate $\int_C \frac{\cos(\pi z)}{z^2 - 1} dz$ around a rectangle with vertices at:
(a) $2 \pm i$ and $-2 \pm i$
(b) $-i, 2 - i, 2 + i, i$ [5+5]

OR

- State and Prove Uniqueness theorem. [10]
4. State and prove Schwarz lemma. [10]
 5. Find all the singularities of $f(z)$ and also evaluate the residues at each of the singularity. [10]
(a) $f(z) = \frac{1}{z^2 + z^4}$
(b) $f(z) = \operatorname{cosec}(z)$ [5+5]
 6. Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent series valid for $1 < |z| < 3$ and $|z| > 3$. [10]
 7. Evaluate $\int_0^\infty \frac{dx}{x^6 + 1}$.