ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU- 27
M.Sc MATHEMATICS- IV SEMESTER
SEMESTER EXAMINATION: APRIL 2023
(Examination conducted in May 2023)
MT 0122: ADVANCED GRAPH THEORY
(For current batch students only)

Duration: 2.5 Hours   **Max. Marks:** 70

1. This paper contains **TWO** printed page.

2. Answer any **SEVEN FULL** questions.

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1. Prove that, if the line graph G has none of the nine forbidden subgraphs as an induced subgraph, then G does not have $K_{1,3}$ as an induced subgraph and if two odd triangles have a common line, then the subgraph induced by their points is K_4. **[10 marks]**

2. (a) Prove that a graph is the line graph of a tree if and only if it is a connected block graph in which each cut point is on exactly two blocks. **[5 marks]**

    (b) Define total graphs. Find the total graph of K_4 and K_5. **[5 marks]**

3. (a) State and prove Euler's formula for planar graphs. **[6 marks]**

    (b) Show that there exists a graph of order $n \geq 3$ and size $m > 3n - 6$ that contains neither K_5 nor $K_{3,3}$ as a subgraph. **[4 marks]**

4. (a) State Jordan curve theorem and show that K_5 is non-planar using Jordan curve theorem. **[6 marks]**

    (b) Is Petersen graph $G(10, 15)$ planar? Justify your answer. **[4 marks]**

5. Prove that a graph has a dual if and only if it is planar. **[10 marks]**

6. If G is a connected vertex-transitive graph, then prove that $\lambda(G) = \delta(G)$. **[10 marks]**

7. (a) Define hypercubes using binary sequence and using cartesian product. Draw hypercubes Q_2, Q_3 and Q_4. **[5 marks]**

    (b) Prove that, for any given vertex x of hypercube Q_n, there exists the unique vertex y such that the distance $d(Q_n; x, y) = n$. Also prove that, there are n internally disjoint (x, y)-paths of length n. **[5 marks]**

8. Prove that the cartesian, the direct, the lexicographic, and the strong product are each associative.

    [10 marks]

9. Prove the distance formula for the cartesian product of graphs. **[10 marks]**

or

Prove the distance formula for the strong product of graphs. **[10 marks]**

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