Register Number:

Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE –560 027

M.Sc. STATISTICS – IV SEMESTER

SEMESTER EXAMINATION – May 2023

ST 0120: Advanced Statistical Inference

Time: 2 ½ hrs **Max: 70 Marks**

This question paper has **TWO** printed pages and **TWO** sections

SECTION – A

I Answer any SIX of the following: 6x 3= 18

1. Define consistent estimator. State and prove invariance property of consistent estimator.
2. If $X_1, X_2, \dots, X_n \sim Poisson(\theta)$, then show that if $\bar{X}$ is Consistent asymptotically Normal (CAN) for θ then $\psi(\bar{X})$ is CAN for $e^{-\theta}$.
3. Write note on Resampling methods. Explain any two applications of resampling methods.
4. Define Sequential Probability Ratio Test (SPRT) and stopping time.
5. Obtain approximations to stopping bounds in SPRT.
6. Explain the procedure of constructing SPRT to test for mean when samples are from normal distribution.
7. Define U-Statistics. State two sample U-statistics theorem.
8. Find the distribution of D_2, the one sample Kolmogorov-Smirnov statistic based on 2 observations.

SECTION – B

II Answer any FOUR of the following: 4 x 13 = 52

9. A) Define CAN and Best asymptotically Normal (BAN) estimators. Give example consistent but not CAN.
   B) State and prove invariance property of CAN estimators. (6+7)

10. A) If $X_1, X_2, \dots, X_n$ be a random sample from $N(0, \sigma^2)$, obtain CAN estimator of σ^2.
    B) Define asymptotic relative efficiency (ARE) of estimators. Obtain efficiency of mle of p when $X_1, X_2,, \dots, X_n \sim Bernoulli(p)$. (7+6)

11. A) Define Huber estimator. Obtain its limiting distribution when $X_1, X_2, \dots, X_n \sim f(x, \theta)$, where f is symmetric about θ.

B) State and Prove Wald's Identity.

C) Define OC and ASN curve. Mention one application of each. (5+5+3)

12. A) Obtain SPRT test for testing the parameter μ, if the samples are drawn from $N(\mu, 1)$.

B) Obtain stopping time bounds for SPRT with an illustration. (7+6)

13. A) Derive the asymptotic variance of a U-statistics in one-sample problem.

B) Define Wilcoxon signed rank test for one-sample problem. Derive null distribution of Wilcoxon signed rank test for one-sample problem under the null hypothesis for testing location parameter. (6+7)

14. A) Define Kolmogorov-Simrnov test for one-sample problem and derive its distribution under the null hypothesis.

B) Describe Seigel-Tukey test for scale problem. Obtain the null distribution of the Statistic. (7+6)

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