ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
M.A. ECONOMICS- I SEMESTER
SEMESTER EXAMINATION: OCTOBER 2019
EC 7418: MATHEMATICAL METHODS FOR ECONOMISTS

This question paper has 2 printed pages and 3 parts

Time: 2.5 Hours   **Maximum Marks-70**

Part A: Answer any 5 of the following questions   **2X5 = 10**

1. Find the total derivative dz/dt, given $z = x^2 - 8xy - y^3$, where $x = 3t$ and $y = 1 - t$.
2. Find the inverse of the matrix
$$A = \begin{pmatrix} 2 & 4 & 5 \\ 0 & 3 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$
3. Given $y = 3x^3 - 6$, examine the geometric shape of the function, if $x>0$.
4. Which investment earns more interest: Rs. 10000 at 6.5% interest compounded quarterly for twenty years, or Rs. 10000 at 5.5% interest compounded continuously for 15 years? Explain.
5. Evaluate the indefinite integral of $\int(x^2 + 2x + 1)dx$
6. Obtain dy/dx for $y^3 + 4x^3 + 2y^2x + 3yx^2 = 0$.
7. Define Nash equilibrium.

Part B. Answer any three of the following: **10 X 3 = 30**

8. Solve for x_1, x_2 and x_3 using Cramer's rule in the system of equations denoted by:
$$x_1 + x_2 + x_3 = 0$$
$$12x_1 + 2x_2 - 3x_3 = 5$$
$$3x_1 + 4x_2 + x_3 = -4$$

9. a) A consumer has the utility function $U = x^\alpha y^\beta$, such that $0<\alpha<1$ and $0<\beta<1$. Show that there is diminishing marginal utility to increased consumption of either commodity.
b) The demand curves of commodities x and y are given by: $P_x = 6 - 0.8q_x$ and $P_y = 6 - 0.4q_y$ respectively. Show that at same price ($P_x = P_y = 2$), the two curves have the same elasticity of demand.
10. Given $Q = AK^\alpha L^{1-\alpha}$, check homogeneity of the function, verify Euler's theorem and calculate elasticity of substitution.

11. a) Check Walras and Marshall stability condition for the given system of equations:
$Q_d = 4 - 2P$ and $Q_s = 2 + 2P$.
b) Consider a multiple-plant monopolist who produces two products x_1 and x_2, whose revenue function is given by $R = 50x_1 + 500x_2 - x_1^2 - x_2^2 - x_1x_2$ and the two cost functions are $C_1 = 3x_1^2 + 33$ and $C_2 = 4x_2^2 + 44$. Find the maximum profit and the quantities that the firm can make.

12. Given the following demand and cost functions $P = 250 - 3q$ and $C = 3q + 5q^2$ respectively, find the profit maximising price and output. How would the firm adjust its price and output, if a tax of Rs. 4 per unit of output be imposed on the firm?

Part C. Answer any two of the following: 15 X 2 = 30

13. a) Let the technological coefficient matrix (A) and the final demand vector (D) are given by:

The coefficient matrix, $A = \begin{matrix} 0.2 & 0.3 & 0.2 \\ 0.4 & 0.1 & 0.2 \\ 0.1 & 0.3 & 0.2 \end{matrix}$ and the final demand vector, $D = \begin{matrix} 10 \\ 5 \\ 6 \end{matrix}$

Find the corresponding output levels of three industries.

b) The MR function is given by $R'(x) = 28 - 15x + 2x^2$. Find the TR function and the demand function.

14. a) A firm produces two products X and Y. The profits per unit of X and Y are Rs. 5 and Rs. 6 respectively. Each product passes through two processes. Product X requires 1 hour of process-I and 2 hours of process-II per unit. Product Y requires 1 hour of process-I and 3 hours of process-II per unit. The firm has a capacity of 5 hours of process-I and 12 hours of process-II. Determine the optimum product-mix of the firm if the objective of the firm is to maximise total profit.
b) Solve the game whose payoff matrix is given by:

$$
\begin{array}{c c}
 & \text{Player B} \\
\text{Player A} &
\begin{array}{c|cccc}
 & B1 & B2 & B3 & B4 \\
A1 & 3 & -1 & 4 & 2 \\
A2 & -1 & -3 & -7 & 0 \\
A3 & 4 & -6 & 2 & -9 \\
\end{array}
\end{array}
$$

15. a) If the production function is of the form $q = 8x_1^{1/2} + 20x_2^{1/2}$ and if $r_1 = 1$, $r_2 = 5$, derive the equation of the expansion path.
b) Assume that the market demand is $P = 100 - 0.5X$ and the two colluding firms have costs given by $C_1 = 5X_1$ and $C_2 = 0.5X_2^2$. Find the cartel quantities and profit.