Register Number:
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ST. JOSEPH'S UNIVERSITY, BANGALORE-27
M.Sc. PHYSICS – II SEMESTER
SEMESTER EXAMINATION – APRIL 2023
(Examination conducted in May 2023)
PH8121 : ELECTRODYNAMICS

Time: 2 hours **Maximum Marks:50**

This question paper contains 2 parts and 3 printed pages.

Some useful Identities:

$$\vec{\nabla}.(f\vec{A})=f(\vec{\nabla}.\vec{A})+\vec{A}.(\vec{\nabla}f)$$

$$\vec{\nabla}.(\vec{A}\,x\,\vec{B})=\vec{B}.(\vec{\nabla}\,X\,\vec{A})-\vec{A}.(\vec{\nabla}\,X\,\vec{B})$$

In Spherical polar co-ordinates

$$\nabla t=\frac{\partial t}{\partial r}\hat{r}+\frac{1}{r}\frac{\partial t}{\partial\theta}\hat{\theta}+\frac{1}{r\sin\theta}\frac{\partial t}{\partial\phi}\hat{\phi}$$

$$\nabla.\vec{v}=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r)+\frac{1}{rsin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(v_\phi)$$

$$\nabla\,x\,\mathbf{v}=\frac{1}{r\sin\theta}[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi)-\frac{\partial v_\theta}{\partial\phi}]\hat{r}+\frac{1}{r}[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi}-\frac{\partial(r v_\phi)}{\partial r}]\hat{\theta}+\frac{1}{r}[\frac{\partial(r v_\theta)}{\partial r}-\frac{\partial v_r}{\partial\theta}]\hat{\phi}$$

In Cylindrical co-ordinates

$$\nabla\,x\,\mathbf{v}=\frac{1}{s}[\frac{\partial v_z}{\partial\phi}-\frac{\partial v_\phi}{\partial z}]\hat{s}+[\frac{\partial v_s}{\partial z}-\frac{\partial(v_z)}{\partial s}]\hat{\phi}+\frac{1}{s}[\frac{\partial(s v_\phi)}{\partial s}-\frac{\partial v_s}{\partial\phi}]\hat{z}$$

All bold letters also denote vectors where ever relevant.

Part-A

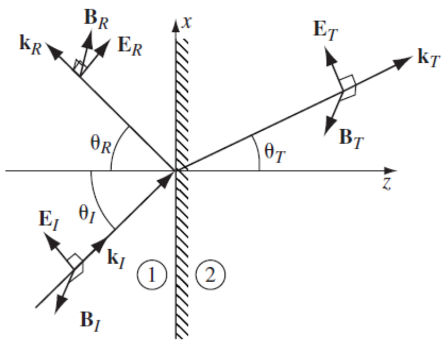
Answer any 5 questions. Each question carries 7 marks. (7x5=35)

1. In general the electric potential of an arbitrary localized point charge at some distance $\vec{r}$ from origin is given as : $V(\vec{r})=1/(4\pi\epsilon_o)\int(\frac{\rho}{\gamma})(d\tau')$ where $d\tau'$ is the elemental volume of this localized charge distribution whose distance from the origin is $\vec{r}\,'$ and distance from the distribution to the far-off point where potential is being determined is $\vec{\gamma}$. Now develop a multipole expansion for the precise potential of this charge distribution at a far off point from the source.

2. a)Write down Maxwell's equations in their general form and derive them in their integral form.
b) What is the significance of Maxwell's equations in general. (5+2)

3. Evaluate the work done on the charges by the electromagnetic force in an instant of time 'dt' in a given configuration of charges and currents and hence, derive Work-energy or Poynting theorem. State what is Poynting vector.

4. Suppose an x-y plane forms the boundary between two linear dielectric media at z=0. An incoming monochromatic plane wave of frequency 'ω', travelling in z-direction, polarized in the plane of incidence (x-z plane) meets the boundary at an angle θ_I as shown in fig. below. It gives rise to reflected wave at angle θ_R and transmitted wave at angle θ_T where $\theta_T < \theta_I$ as velocity of wave in medium 2 v_2 is less than than in medium 1 v_1. Assume that all the three laws of geometrical optics are obeyed. Show that the Fresnel's equations for this polarization state are $\tilde{E}_{0R} = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)\tilde{E}_{0I}$ and $\tilde{E}_{0T} = \left(\frac{2}{\alpha+\beta}\right)\tilde{E}_{0I}$ where $\tilde{E}_{0I}, \tilde{E}_{0R}, \tilde{E}_{0T}$ are the incident, reflected and transmitted amplitudes. Here $\alpha = \frac{\cos\theta_T}{\cos\theta_I}$ And $\beta = \frac{\mu_1 v_1}{\mu_2 v_2}$ Assume $\mu_1 \approx \mu_2 \approx \mu_0$. What is Brewster's angle?



5. a) Reformulate Maxwell's equations in terms of the potentials $V, \vec{A}$ for the time-dependent configuration.
b) Now, use Lorentz gauge to simplify them. What is the advantage of this gauge?(3+4)

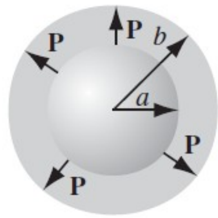
6. Consider an oscillating dipole made up of two tiny charged metal spheres with charge +q(t) and –q(t) separated by a distance 'd' oscillating with angular frequency 'ω'. The potentials at a point 'P' at time 't' in the far radiation zone (d<<λ<<r) are given as
$V(r,\theta,t) = \frac{-p_o\omega}{4\pi\epsilon_o c}\left(\frac{\cos\theta}{r}\right)\sin\left[\omega\left(t-\frac{r}{c}\right)\right]$ $\vec{A}(r,\theta,t) = \frac{-\mu_o p_o \omega}{4\pi r}\sin\left[\omega\left(t-\frac{r}{c}\right)\right]\hat{z}$ where $\hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$, 'r' is the distance from centre of dipole to a point 'P' and θ is the acute angle between 'd' and 'r'. Find the fields $\vec{E}, \vec{B}$ at this point 'P' and the intensity of radiation radiated by the dipole.

7. Suppose you had a string of +ve charges moving along to the right at speed 'v' and superimposed on this +ve string is a -ve string proceeding to the left with the same speed 'v' such that I=2λv in reference frame S.(Assume that the charges are close enough so that linear charge density is λ. Also, the line charge density in the rest frame of charges is λ_o). Now, suppose there is a charge +q at a vertical distance 's' away from this assembly which moves with speed 'u' such that u<v. The net electrical force on this charge in this frame S is zero.
Now, analyse the system from another frame $\bar{S}$ which is moving to the right at a speed 'u' and find the force experienced by this charge in this frame $\bar{S}$. Using the transformation rules for forces i.e. $\bar{F}_\perp = \frac{F_\perp}{\gamma}$ where $F_\perp$ is the force in the frame in which the speed of particle is zero and $\gamma = \frac{1}{\sqrt{1-u^2/c^2}}$, find the force experienced by the charge q in the frame S. Discuss what is the nature of the force in frame S.

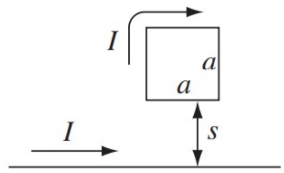
Part-B

Answer any 3 questions. Each question carries 5 marks. **(3x5=15)**

8. A thick spherical shell of inner radius a and outer radius b is made of dielectric material with a "frozen-in" polarization $\vec{P}(\vec{r})=\frac{k}{r}\hat{r}$ as shown in figure, where k is a constant and r is the distance from the centre. (There is no free charge in the problem.) Find the electric field in all three regions: r<a, a<r<b, r>b.



9. A square loop is placed near an infinite straight wire as shown in the figure below. Both the loop and the wire carry current 'I' as shown. Find the direction and magnitude of the net magnetic force acting on this loop.



10. The electric and magnetic fields in a conductor have plane wave solutions where the wave number $\tilde{k}$ is a complex quantity and is given as $\tilde{k}=k+i\kappa$ where $k=\omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{\left(1+\frac{\sigma}{\epsilon\omega}\right)}+1\right]^{(1/2)}$ $\kappa=\omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{\left(1+\frac{\sigma}{\epsilon\omega}\right)}-1\right]^{(1/2)}$

a) Show that in a good conductor, the magnetic field lags the electric field by 45^0 .

b) The skin depth in poor conductors is $\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$ and in good conductors is $\sqrt{\frac{2}{\omega\mu\sigma}}$.

(2+3)

11. An infinite straight wire carries a current $I(t)=I_o$ as shown in the figure which is turned on abruptly at t=0. Sine the wire is electrically neutral so, the scalar potential is zero. Show that the retarded vector potential $\vec{A}$ is given as $\vec{A}(s,t)=\frac{\mu_o I_o}{2\pi}\ln\left(\frac{ct+\sqrt{(ct)^2-s^2}}{s}\right)\hat{\mathbf{z}}$ and find the resulting fields $\vec{E},\vec{B}$.