Registration Number:

Date & session:

ST.JOSEPH'S UNIVERSITY, BENGALURU -27

M.Sc (PHYSICS) – II SEMESTER
SEMESTER EXAMINATION: APRIL 2023
(Examination conducted in May 2023)
PH 8321 – STATISTICAL PHYSICS
(For current batch students only)

Time: 2 Hours **Max Marks: 50**

This paper contains 4 printed pages and 2 parts

PART A

Answer any FIVE full questions. **(5x7=35)**

1.
   (a) Write down the Hamiltonian for a particle making up an ideal monatomic gas.
   (b) Assuming no interactions between particles, compute the partition function of the system assuming it to be a canonical ensemble. Use classical approach to derive the partition function and assume that the particles are indistinguishable.
   (c) What is the mean energy of this ensemble? [1+4+2]

2.
   (a) What is exchange degeneracy?
   (b) Define the exchange operator and show that such degeneracies give rise to two types of particles quantum mechanically. [1+6]

3. Consider a quantum system made up of a total of N particles. Each microstate energy level in the system labeled with the index r may have n_r occupancies.
   (a) Write down the partition function for this system assuming a canonical distribution.
   (b) From this, write down the probability distribution function.
   (c) Estimate the mean occupancy $\overline{n_s}$ in terms of the partition function. [1+2+4]

4.
   (a) In two sentences, summarize the Riemann-Zeta function of order 4.
   (b) Given that the mean energy density for photons confined to a cavity and in thermal equilibrium with itself per unit frequency is given as: $\bar{u}(\nu;T)=\frac{8\pi h}{c^3}\left(\frac{\nu^3}{e^{\frac{h\nu}{k_B T}}-1}\right)$:
      i. Obtain the total mean energy density summed over all frequencies.
      ii. While doing so, establish the connection with the Riemann-Zeta function of order 4.

   [2+4+1]

5.

(a) For an Ideal Bose Gas, the mean occupancy of a microstate labeled by s is: $\bar{n}_s = \frac{1}{e^{\beta(\epsilon_s - \mu)} - 1}$, with ϵ_s being the microstate energy and μ being the chemical potential. What is the condition on the values of ϵ_s and μ ?

(b) What is fugacity?

(c) Using the expression from part (a) and the definition of fugacity from part (b) above, express the natural logarithm of the total partition function for a Bose-Einstein Gas in terms of fugacity and energy. [1+1+5]

6.

(a) What is the mean occupancy of a fermion in an Ideal Fermi Gas?

(b) What is the condition on the microstate energy of a fermion in this system and how is it related to its chemical potential?

(c) What is the asymptotic limits of the mean occupancy at $T = 0\,\mathrm{K}$? [2+2+3]

7.

(a) What is the mean energy of a Fermi Gas at $T = 0\,\mathrm{K}$?

(b) Using this expression find out the mean pressure of the gas $\left(\bar{P} = -\frac{\partial \bar{E}}{\partial V}\right)$. [3+4]

PART-B

Answer any THREE full questions **(3x5=15)**

[Constants: h=6.6x10^{-34} J s (Planck's constant), 1eV = 1.6x10^{-19} J (electron volt to Joules), c=2.99x10^{8} m/s (speed of light), 1Å = 1x10^{-10}m (Angstrom to meters), k_B = 1.380649x10^{-23} JK^{-1} (Boltzmann constant), N_A= 6.022 × 10^{23} mole^{-1} (Avogadro Number), e = 1.6x10^{-19} C (electronic charge), m_{proton}=1.673x10^{-27}kg (mass of proton), $m_{electron}$=9.109x10^{-31}kg (mass of electron), G=6.674x10^{-11}m^{3}kg^{-1}s^{-2} (Gravitational constant), $M_\odot$=1.9891x10^{30} kg (Solar mass), $R_\odot$=6.9x10^{8} m, σ=5.67x10^{-8}Wm^{-2}K^{-4}(Stefan-Boltzmann constant), M_{Earth}=5.97x10^{27}kg (Mass of Earth), $D_{earth-sun}$=1.49x10^{11}m (Earth-Sun distance), 1 inch = 2.54 cm, 1AU= 1.496x10^{11} m, 1 ly= 9.461x10^{15} m, 1 pc= 3.086x10^{16}m]

8. (i) The volume of an ideal gas is doubled (i.e. V is changed to V' such that $V' = 2V$) keeping its pressure P constant. Estimate the factor by which the total accessible states of the system would have changed (i.e. compute $\frac{\Omega(E', V')}{\Omega(E, V)}$) - the number of particles constituting the gas remains constant.

--OR--

(ii) A container holds a gas of molecules having mass m and is maintained at a constant temperature T. If $\vec{v}$ is the velocity of the molecule such that v_x, v_y and v_z are its components along the x, y and z axis, what are the following mean values:

(a) $\overline{v_x}$ (b) $\overline{v_z^2}$ (c) $\overline{v_x v_y^2}$ (d) $\overline{v_x^2+v_z^2}$ (e) $\overline{(v_x+a v_z)^2}$ with a being a constant

[1+1+1+1+1]

9. (i) A system is made up of a very large number N of particles that do not interact with each other and occupy fixed positions (like in a lattice). Each particle can be in any two possible microstates having energies ϵ_1 or ϵ_2. Of the N particles, at any given time, n_1 can be in the state having energy ϵ_1 and n_2 in the state having energy ϵ_2. The total number of particles remain conserved throughout, however particles in general keep transitioning between the two states in such a manner that the total energy remains between E and $E+\delta E$. Write down the total number of accessible states of the system in the quantum limit (i.e. distinct states) treating the particles as indistinguishable and from this, infer the expression for the entropy of the system. Use Stirling's approximation in its simplest form and simplify the expression for the entropy.

--OR--

(ii) We have seen that the Maxwell Distribution of Speed for an ideal gas (made up of atoms or molecules) is given as: $F(v)\ dv = 4\pi n\left(\frac{m\beta}{2\pi}\right)^{\frac{3}{2}} v^2 e^{-\frac{1}{2}\beta m v^2} dv$ which represents the man number of particles (atoms or molecules) per unit volume with a speed between v and $v+dv$. Convert this distribution to that of energy: $F(\epsilon)\ d\epsilon$ with $\epsilon=\frac{1}{2}m v^2$ (so that now, the distribution will represent the mean number of particles per unit volume with energies between ϵ and $\epsilon+d\epsilon$.

10. (i) A two level system is in thermal equilibrium with a heat reservoir of temperature T. The lower energy level has an energy of $0\,eV$ while the upper level has $0.3\,eV$. There is a 30% probability that the system is in the upper level. What is the temperature of the heat reservoir?

--OR--

(ii) A "non-ideal" gas has its molecules interacting with each other with an attractive potential directly proportional to the **first power of distance** between them. Express the total energy classically. If the gas is in thermal equilibrium at a temperature T, what will the mean energy of the particles of this gas be?

11.

(a)

i. Write down that Hamiltonian of a particle in a simple harmonic oscillator potential.

ii. If the particle is on a cart that it moving with a velocity v_c, what will its mean energy in the classical limit be (you may use the equipartition theorem for this – there is no need for any lengthy derivation)? [1+2]

(b) If ζ represents the single particle partition function for a particle in a monatomic gas, containing N particles that are distinguishable and non-interacting:

i. Write down the total partition function of the gas (assume N to be very large).

ii. From the partition function (assuming canonical distribution), find the chemical potential of the gas (hint: you will need to work out the Helmholtz free energy of the system) [0.5+1.5]