Registration No.:
Date and Session::

ST.JOSEPH'S UNIVERSITY, BENGALURU -27
M.Sc. (PHYSICS) – II SEMESTER

SEMESTER EXAMINATION: APRIL 2023

(Examination Conducted in May 2023)

PH 8421-Quantum Mechanics-I

Time: 2 **hours** **Maximum marks: 50**

This question paper has 2 printed pages and 2 parts

PART A

Answer any **FIVE** of the following questions. Each question carries 7 marks. $[5 \times 7 = 35]$

1. (a) Calculate the expectation value of position and momentum of the nth stationary state of an infinite square well potential. [4]

   (b) For an infinite potential well, show that for each energy eigenvalue, there exists two momentum states. Explain. [3]

2. Solve the azimuthal part of Schrodinger's wave equation for the hydrogen atom using spherical polar coordinates and arrive at the associated Legendre function. [7]

3. Solve the radial equation of the hydrogen atom and arrive at the spherical Bessel function and spherical Neumann function of the order l where l is the azimuthal quantum number. [7]

4. (a) Using the Rodrigues formula obtain the first three Hermite polynomials for a quantum harmonic oscillator. [4]

   (b) Prove that conservation of distinguishable states, i.e., two states are distinguishable if they are orthogonal, implies that the time evolution of states have to be unitary. [3]

5. (a) How does the expectation value of an operator change with respect to time according to Schrodinger picture? [4]

   (b) Evaluate $\frac{d\langle x \rangle}{dt}$, given that the potential is independent of time. [3]

6. Prove that $[b, b^\dagger] = 1$ and hence demonstrate that the Hamiltonian $H = \frac{\hbar\omega}{2}(bb^\dagger + b^\dagger b)$ can be reduced to $H = \hbar\omega(bb^\dagger + 1/2)$ Given:-$b = (\frac{m\omega}{2\hbar})^{1/2}(x + \frac{ip}{m\omega})$, $b^\dagger = (\frac{m\omega}{2\hbar})^{1/2}(x - \frac{ip}{m\omega})$ where x, p are the position and momentum operators respectively. [7]

7. If the total angular momentum $\vec{J} = \vec{J_1} + \vec{J_2}$ and if it satisfies the commutation relations, given in short form as $[J_k, J_l] = i\hbar\epsilon_{k,l,m}J_m$, show that $[J^2, J_z] = 0$ and $[J^2, J_1^2] = 0$ [7]

PART B

Answer any **THREE** of the following questions. Each question carries 5 marks. [$3 \times 5 = 15$]

8. A particle of mass m moves in a three-dimensional box of sides a, b, c. If the potential is zero inside and infinity outside the box, find the energy eigenvalues and eigenfunctions

9. A particle of mass m confined to move in a potential $V(x) = 0$ for $0 \leq x \leq a$ and $V(x) = \infty$ otherwise. The wave function of the particle at time $t = 0$ is $\psi(0) = A(2\sin\left(\frac{\pi x}{a}\right) + \sin\left(\frac{3\pi x}{a}\right))$. Normalise the wavefunction.

10. If $b\psi_0 = 0$, where ψ_0 is the ground state of the quantum harmonic oscillator, and $b = (\frac{m\omega}{2\hbar})^{1/2}(x + \frac{ip}{m\omega})$, the annihilation operator, calculate the normalized value of ψ_0.

11. If $j = \frac{1}{2}$, what are the possible values of m?. Find the matrix form of J_+ and J_- operators.