Register number:

Date and session:

ST. JOSEPH'S UNIVERSITY, BENGALURU-27
UG OPEN ELECTIVE - III SEMESTER
SEMESTER EXAMINATION: OCTOBER, 2023
(Examination conducted in November/December 2023)

MTOE 9: MATHEMATICS FOR LIFE SCIENCES II

(For current batch students only)

Duration: 2 Hours **Max. Marks:** 60

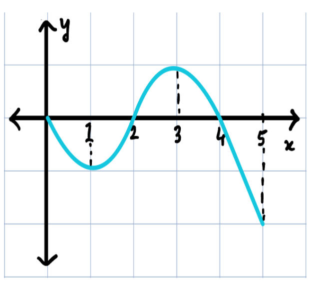
1. The paper contains **TWO** printed pages and **THREE** parts.
2. Scientific calculators are allowed.

Part A: Answer any 6

1. Write the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^4 + \left(\frac{d^3y}{dx^3}\right)^3 + y = 0.$ [2]

2. Solve the differential equation $\frac{dy}{dx} = \frac{x}{y}$. [2]

3. Which of the following statements is true about the graph of the function drawn below? [2]



(i) The function has a local minimum at 1 which is a global minimum.

(ii) The function has a local minimum at 1 which is not a global minimum.

(iii) The function has a local maximum at 3 which is a global maximum.

(iv) The function has a local maximum at 3 which is not a global maximum.

4. Let $z = \ln(x^2 + y^2)$. Compute the first order partial derivatives of z. [2]

5. Show that $(-2, 0)$ is a critical point for the function $f(x, y) = x^3 + 6xy^2 - 2y^3 - 12x$. [2]

6. Write down the recurrence relation for the Fibonacci sequence along with the initial conditions. [2]

7. In the Lotka Volterra predator-prey model if the prey have a place of refuge that can accommodate k prey then write down the new differential equations obtained. [2]

8. Define SIS model for spread of infectious disease. [2]

Part B: Answer any 3

9. Solve the differential equation $\dfrac{dy}{dx} = x^2y - 2x^2 - 3xy + 6x - 9y + 18$ by variable separable method. [6]

10. Derive the differential equation that arises from a birth-death process and solve it. You may assume a constant proportion of reproductive individuals in the population, constant fertility, plentiful resources and no immigration/migration. [6]

11. A square sheet of cardboard with each side 12cm is to be used to make an open top box by cutting out a small square from each corner and bending up the sides. What is the side length of the small square if the box must have maximum volume? [6]

12. Find the critical points of the function $f(x) = x^2e^{2x}$ and determine whether they are local maxima, local minima or saddle points. [6]

13. Sketch the graph of the function $f(x) = x^3 - 3x$ in the range $[-3, 3]$. You may assume the function has a local maxima at $x = -1$ and a local minima at $x = 1$. [6]

Part C: Answer any 5

14. Find all the second order partial derivatives of the function $z = \cos(x^2 + y^2)$. Also show that $\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{\partial^2 z}{\partial y \partial x}$. [6]

15. Find all the critical points of the function $z = 4y^3 - 3y^2x + x^3 - 9x$. [6]

16. Examine the nature of the critical points $(1, -2)$ and $(-1, -2)$ of the function, $z = x^3 + y^2 - 3x - 12y + 2$. [6]

17. Find the critical points of the function $f = 2x^2+2y^2+4z^2$ subject to the constraint $x+y+z = 1$. [6]

18. Explain contest competition and scramble competition in insect population dynamics. [6]

19. Describe the Lotka Volterra predator prey model and solve the differential equations obtained. [6]

20. Explain the SIR model for infectious disease spread. [6]