Registration Number:

Date & Session

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU -27

B. Sc – V SEMESTER

SEMESTER EXAMINATION: OCTOBER 2023

(Examination conducted in December 2023)

ST 5123: Statistical Inference II

Time: 2 Hours **Max Marks: 60**

This paper contains TWO printed pages and THREE parts.

Usage of a scientific calculator is permitted.

PART-A

I. Answer any FIVE from the following **3 * 5 = 15**

1. Define Most Powerful (MP) test. Let the random variable $X_i \sim N(0, \sigma^2)\ i = 1,2, \dots n$. For testing the hypothesis $H_0: \sigma^2 = \sigma_0^2$ against $H_1: \sigma^2 = \sigma_1^2 (> \sigma_0^2)$ at level α the MP test is given by $\phi(x) = \begin{cases} 1\ if\ \sum x_i^2 > \sigma_0^2 \chi_{\alpha,n}^2 \\ 0\ if\ \sum x_i^2 \leq \sigma_0^2 \chi_{\alpha,n}^2 \end{cases}$. Give the power of the test.
2. State the relation between uniformly most powerful (UMP) test and Monotone Likelihood ratio (MLR) property.
3. Let $X \sim N(\mu, \sigma_0^2)$. Give the Likelihood ratio (LR) test function for testing following hypothesis:
   i. $H_0: \mu = \mu_0\ against\ H_1: \mu > \mu_0$ ii. $H_0: \mu = \mu_0\ against\ H_1: \mu < \mu_0$
4. List out the assumptions of paired t test.
5. Write the different uses of χ^2 distribution.
6. Mention any of the three distribution-free tests.
7. What is a run test in statistics, and why is it used?

PART - B

II. Answer any FIVE from the following **5 * 5 = 25**

8. Let $X_1, X_2, \dots X_n$ be a random sample from $N(\mu, 1)$. For testing $H_0: \mu = \mu_0$ against $H_1: \mu = \mu_1 (< \mu_0)$. Obtain an MP test at level α.
9. Show that $N(\mu, \sigma_0^2)$ possess the MLR property.
10. Let $X_1, X_2, \dots X_n$ be a random sample from B (1, p). For testing $H_0: p = p_0$ against $H_1: p < p_0$. Construct the UMP test at level α.

11. Describe the test of significance for the regression coefficient for small samples.
12. Describe the steps involved in conducting a hypothesis test for the difference in means between two independent groups using a t – test when two groups have unknown but equal variance. Give specifics on framing hypotheses, calculating test statistic, and interpreting results.
13. Explain the test of significance for attributes for large samples.
14. Describe the procedure of Kolmogorov-Smirnov one sample test.

PART - C

III. Answer any TWO from the following **10 * 2 = 20**

15. A) Describe the test for ratio of two variances.

    B) Let $X_1, X_2, \ldots X_k$ be a random sample from B (m, θ). For testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1 (> \theta_0)$. Establish the MP test at level α. (4+6)

16. A) Define (i) Parametric Space. (ii) LR test.

    B) Let $X_1, X_2, \ldots X_m$ be a random sample from $N(\mu, \sigma^2)$. For testing $H_0: \sigma = \sigma_0$ against $H_1: \sigma \neq \sigma_0$. Derive the LR test at level α. (3+7)

17. A) Briefly compare the Wilcoxon signed-rank test and the paired t-test. When would you opt for the Wilcoxon signed-rank test in an analysis involving paired data?

    B) Describe the Median test (5+5)

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