Registration No.:
Date and Session::

ST.JOSEPH'S UNIVERSITY, BENGALURU -27
M.Sc. (PHYSICS) – I SEMESTER
SEMESTER EXAMINATION: October 2023
(Examination Conducted in November/December 2023)

PH 7221-MATHEMATICAL PHYSICS

(Current batch of students only)

Time: 2 **hours** **Maximum marks: 50**

This question paper has 1 printed pages and 2 parts

PART A

Answer any **FIVE** of the following questions. Each question carries 7 marks. $[5 \times 7 = 35]$

1. (a) Write all possible equations of transformation for a mixed tensor of rank four [4]
   (b) If A^{ij}_{kl} is a tensor, prove that a double contraction yields an invariant [3]
2. (a) Using the Method of Separation of Variables solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-3x}$ [7]
3. (a) Express $f(x) = 4x^3 + 6x^2 + 7x + 2$ in terms of Legendre Polynomials. [4]
   (b) Using Method of Characteristics find the general solution of the quasilinear Partial Differential Equation $a\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = 0$ [3]
4. (a) State Bessel's differential equation. Write Spherical Bessel Functions. [4]
   (b) What are Cauchy-Riemann conditions and why are they important in complex analysis? [3]
5. (a) State and explain Cauchy's theorem. [3]
   (b) Calculate the contour integral of $f(z) = z^2$ along the unit circle $|z| = 1$ in the counter clockwise direction and show that it satisfies Cauchy's theorem. [4]
6. (a) Explain the concept of frequency domain and time domain in the context of Fourier transforms. [3]
   (b) Describe the properties of 1) Linearity and 2) time-shifting with reference to Fourier transforms. [4]
7. (a) Find the Fourier Transform of a Gaussian function. [5]
   (b) How does the choice of a windowing function such as a rectangular function affect the Fourier transform of a function? [2]

PART B

Solve any **THREE** of the following problems. Each problem carries 5 marks. $[3 \times 5 = 15]$

8. A rectangular plate bounded by the lines x=0, x=a; y=0, y=b has an initial distribution of temperature given by $u = Asin\frac{\pi x}{a}sin\frac{\pi y}{b}$ the edges are kept at zero temperature and the plane faces are impervious to heat. Determine the temperature distribution at a later time t
9. Prove the recurrence formulae for Hermite Polynomials $H'_n(x) = 2nH_{n-1}(x)$
10. Evaluate $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$ using residue theorem.
11. Find the power spectrum of an exponential decaying function.