Register Number :

Date and Session:

ST. JOSEPH'S UNIVERSITY, BANGALORE-27
M.Sc. (PHYSICS) – III SEMESTER
SEMESTER EXAMINATION – OCTOBER 2023
(Examination conducted in November/December 2023)
PH 9320 : MODERN OPTICS
(For Current Batch Only)

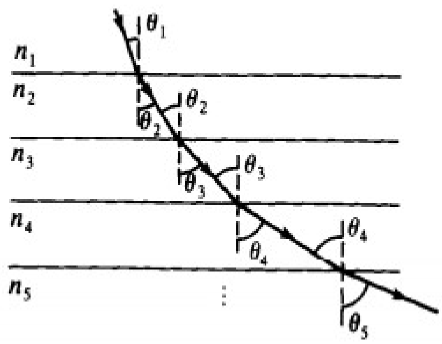
Time: 2 hours **Maximum Marks:50**

This question paper contains 2 parts and 3 printed pages.

Part-A

Answer any **FIVE** questions. Each question carries **7** marks. **(7x5=35)**

1. a) Consider the light rays, which are refracted by a medium comprising a continuous set of thin slices of media of different refractive indices as shown in figure



Obtain the equation that describes the direction of light rays.

b) Write a short note on material dispersion. (5+2)

2. In a double slit Fraunhofer diffraction pattern, the intensity distribution at a point on the screen is given as $I = I_o \frac{\sin^2\beta}{\beta^2}\cos^2\gamma$ where β is given as $\beta = \frac{\pi b \sin\theta}{\lambda}$ and γ is given as $\gamma = \frac{\pi(b+d)\sin\theta}{\lambda}$. Here,'b' is the width of each slit and 'd' is the separation distance between the two slits and θ is the angle that each ray of diffracted light makes with the normal to the grating.

a) Explain the terms in the intensity relation given above. Find the positions of various maximas and minimas. Draw the intensity distribution curve from it for d=2b.

b) Explain how the intensity pattern will get affected if we increase the slit widths keeping the separation distance between them to be the same. (5+2)

3. A monochromatic plane wave propagating in x-direction is incident on an opaque diffracting aperture. According to the Huygen-Fresnel principle, if ϵ_A is the source strength per unit area, assumed to be constant over the entire aperture then the total optical disturbance at a point P (under the far-field approximation) on the screen due to differential area of element dS within the aperture is given as $\tilde{E}=\left(\frac{\epsilon_A}{R}\right)e^{i(\omega t-kR)}\iint\limits_{\text{Aperture}} e^{ik(Yy+Zz)/R}\,dS$ where R is the distance from centre of aperture to the screen point P (X,Y,Z) and $dS=dydz$ .If the aperture is a circular opening with radius 'a' then using polar coordinates for the plane of aperture and the plane of observation find the intensity of the field at point P assuming that the intensity at P_o i.e. the centre of the screen is given as $I(0)=\frac{\epsilon_A^2 A^2}{2R^2}$ where A is the area of circular aperture.

[Given :The Bessel function of (first kind) order zero is given as $J_o(u)=\frac{1}{2\pi}\int_0^{2\pi} e^{i\,u\cos\theta}d\theta$ and the recurrence relation for the Bessel function is $\frac{d}{du}[u^m J_m(u)]=u^m J_{m-1}(u)$]. (7)

4. a) Briefly explain the phenomenon of double refraction as an unpolarised light beam is incident normally on a uniaxial crystal. Also, elaborate on the velocities of o-ray and e-ray.

   b) In a uniaxial crystal, if a plane wave propagates at an angle ψ to the optic axis(z-axis) in the x-z plane inside an anisotropic medium then for extra-ordinary wave, $\frac{K_z E_z}{K_x E_x}=-\tan\psi$ where K_x, K_y, K_z give the principal dielectric constants along the three principal axes x, y, z respectively with propagation vector $\vec{k}=k\hat{\kappa}$ and E_x, E_y, E_z are the components of the electric field in the principal axes system. Find the angle between $\vec{E}$ and $\vec{D}$ for this state of polarisation of the wave and comment on its sign for positive and negative uniaxial crystals. (4+3)

5. a) Derive the inhomogeneous wave equations from Maxwell's equations for non-linear dielectric medium (neglecting the anisotropy of the medium).

   b) Now, if an optical wave of frequency ω travelling in z-direction is incident on a non-linear medium with electric field given as $E^{(\omega)}=E_1\cos(k_1 z-\omega t+\phi_1)$ then express the field in terms of the complex field and its conjugate. Knowing that wave propagation within the non-linear medium generates 2ω frequency and using $P^{NL}=\epsilon_o\chi^{(2)}E^2$, evaluate the non-linear polarisation at frequencies ω and 2ω . (4+3)

6. a) In a non-linear medium capable of showing second order non-linear effects, the non-linear polarisation at 2ω travels with a velocity ω/k_1 and the electromagnetic wave generated at 2ω travels with velocity $2\omega/k_2$. From this, obtain the phase matching condition and explain what it means in terms of refractive indices.

   b) Calculate the coherence length for $LiNbO_3$ crystal if it is illuminated by Nd-YAG laser of wavelength 106 nm. The refractive index of the crystal for extra-ordinary wave at wavelength λ is 2.1561 and at $\lambda/2$ is 2.2355.

   c) Will light pass through if we put a half wave plate at 45° angle in between two linear crossed polarizers? Explain. (3+2+2)

7. The intensity of light at a point on the screen, coming from 2 pin holes which are illuminated by a single source is given as $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\Re(\gamma_{12}(\tau))$ where $\gamma_{12}(\tau) = \frac{\langle E_1(t+\tau)E_2^*(t)\rangle_T}{\sqrt{\langle|E_1^2|\rangle_T|\langle|E_2^2|\rangle_T|}}$ is the complex degree of coherence, subscripts 1 and 2 correspond to the two pinholes and $\tau = \frac{r_2 - r_1}{c}$ where r_2 and r_1 are the respective distances from 1 and 2.
   a) Express the complex degree of coherence in terms of modulus and phase part. From that comment on what the phase part signifies and how do we determine the range of values for the modulus part.
   b) Explain how this complex degree of coherence would change for Michelson's interferometer if the incident field $E(t) = E_o e^{i\phi(t)}$ falls on the interferometer. (4+3)

Part-B

Answer any **THREE** questions. Each question carries **5** marks. **(5x3=15)**

8. In a grating, the Sodium doublet of wavelength 5890Å and 5896 Å is viewed in the third order at 30° to the normal and is found to be just resolved. Find the grating element, the total number of lines required to just resolve the doublet and the total width of the grating.

9. Find the Fourier transform of a Gaussian pulse of light which is given as $\vec{E}(x,0) = f(x)e^{-ik_o x}$ where $f(x) = \sqrt{(\frac{a}{\pi})}e^{-ax^2}$ using convolution theorem.

10. Plane-polarized light of wavelength $5 x 10^{-7}$ m is incident on a quartz plate cut parallel to optic axis. Find the least thickness of the plate for which the ordinary and extraordinary rays combine to form plane-polarized light on emergence. What multiples of this thickness would give the same result? The indices of refraction of quartz are n_e = 1.5533 and n_o = 1.5442.

11. a) A filter passes light with a mean wavelength of $\bar{\lambda}_o = 500\,nm$. If the emerging wave trains are roughly $20\bar{\lambda}_o$ long, then find the linewidth and frequency bandwidth of the source.
    b) Find the Fourier Transform of the function $f(x) = \delta[x - (+d/2)] - \delta[x - (-d/2)]$.
    (2+3)