Registration Number:
Date & session:

ST JOSEPH'S UNIVERSITY, BENGALURU -27
M.Sc. (STATISTICS) – I SEMESTER
SEMESTER EXAMINATION: OCTOBER 2023
(Examination conducted in November /December 2023)
ST 7121: PROBABILITY THEORY
(For current batch students only)

Time: 2 Hours **Max Marks: 50**

This paper contains TWO printed pages and ONE part.

PART-A

I. Answer any FIVE questions out of SEVEN questions:

1. A) Define monotonic sequence of sets with an example.

   B) Define Field. Prove that every σ field is a field.

   C) Define probability measure. Prove that if A and B are independent then A and B^c are independent. (2+4+4)

2. A) Check whether a sequence of independent random variables $\{X_k\}$ satisfies WLLN, where probability distribution of X_k is as follows:

X_k	-2^k	0	2^k
$P(X_k = x)$	$\frac{1}{2^{2k+1}}$	$1 - \frac{1}{2^{2k}}$	$\frac{1}{2^{2k+1}}$

   B) Define convergence in distribution.

   C) Prove that distribution function of a random variable is non-decreasing and right continuous. (4+2+4)

3. A) Prove that convergence in distribution need not imply convergence in probability.

   B) Define the quantile function with an example.

   C) Write a note on decomposition of a distribution function. (5+2+3)

4. A) State and prove Chebyshev's Inequality.
   B) Define expectation of a simple random variable. Prove that E(X±Y)=E(X) ± E(Y).
   (6+4)
5. A) If X and Y are two random variables then prove that E {Min(X, Y)} ≤ Min {E(X), E(Y)}.
   B) State and prove the Inversion theorem of a characteristic function. (2+8)

6. A) Define Moment Generation Function (MGF). State and prove any one property of MGF.
   B) Define Characteristic function. Derive the Characteristic function of Uniform distribution over the interval (a, b).
   C) If $\Phi_X(t)$ is a characteristic function of a random variable X, then prove that $|\Phi_X(t)| \leq 1$.
   (4+4+2)
7. A) Prove that characteristic function is uniformly continuous on **R**.
   B) Mention any one application of MGF and characteristic function.
   C) For a measure μ and events $A_1, A_2, \ldots, A_k$ prove that $\mu\left(\bigcup_{n=1}^{k} A_n\right) \leq \sum_{n=1}^{k} \mu(A_n)$
   (5+1+4)

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