Registration Number:

Date & session:

ST JOSEPH'S UNIVERSITY, BENGALURU -27
M.Sc. (STATISTICS) – III SEMESTER
SEMESTER EXAMINATION: OCTOBER 2023
(Examination conducted in November /December 2023)
ST 9120: STOCHASTIC PROCESSES
(For current batch students only)

Time: 2 Hours **Max Marks: 50**

This paper contains TWO printed pages and ONE part.

PART-A

I. Answer any FIVE questions out of SEVEN questions:

1. A) Define Markov chain and show that the Markov chain $\{X_n, n = 0,1,2,...\}$is completely determined by transition probability matrix 'P' and the initial probability distribution {p_i} defined as $P\{X_0 = i_0\} = p_i \geq 0$.

   B) Let $\{X_n, n = 0,1,2,...\}$be a Markov chain with state space $S = \{1,2,3\}$and with the following TPM

$$P = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0 & 0.3 & 0.7 \\ 0 & 0.2 & 0.8 \end{bmatrix}$$

   Find. (i) $P(X_4 = 2/X_2 = 1)$ (ii) $P(X_5 = 3/X_4 = 3)$

(5+3+2)

2. A) Let $\{X_n, n = 0,1,2,...\}$ be a sequence of i.i.d random variables then, show that $\{Y_n\}$is a Markov chain where, $Y_n = \sum_{i=1}^{n} X_i$.

   B) Define communicating, absorbing, persistent and transient states of the Markov chain.

(5+5)

3. A) Let $\{X_n, n = 0,1,2,...\}$be a Markov chain with state space $S = \{1,2,3,4\}$ and one step

   TPM $P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix}$

   Classify the states as recurrent or transient.

   B) if $i \leftrightarrow j$ and 'i' is persistent then show that 'j' is also persistent. (5+5)

4. A) Explain Mean Recurrence time.

   B) Consider a Gambler's ruin problem with p=0.6, N= 10 and j=4. Find the probability that Player 'A' losing all the amount given that Player 'A' has 4 units of amount.

   C) Find the probability of ultimate extinction if probability generating function of offspring distribution is $\emptyset(s) = \frac{1}{4}s^2 + \frac{5}{8}s + \frac{1}{8}$ (3+3+4)

5. A) Define Poisson process with an example. Prove that for a Poisson process $\{N(t): t > 0\}$ conditional distribution of $N(s)/N(t)$ follows Binomial distribution if $s < t$.

   B) Define renewal function Obtain the renewal equation for the renewal process with Inter arrival time as U (0,1). (5+5)

6. A) Define birth death process.

   B) Prove that in a Poisson process

      (i) Inter arrival time follows exponential

      (ii) Waiting time follows Gamma distribution.

   C) Define branching process. Find the mean and variance of 10th population size if offspring distribution takes values 0,1 and 2 with respective probabilities $\frac{1}{8}$, $\frac{5}{8}$ and $\frac{1}{4}$.

   (1+5+4)

7. A) For a branching process $\{X_n\}$ with $X_0 = 1$, $\emptyset(\,)$ and $\emptyset_n()$ as the probability generating function of ξ_i (off spring distribution) and X_n , prove that

      (i) $\emptyset_{n+1}(s) = \emptyset_n(\emptyset(s))$

      (ii) $\emptyset_{n+1}(s) = \emptyset(\emptyset_n(s))$

   B) Define martingale. For a sequence of independent random variables $\{U_i: i = 1,2,3,..\}$ be each having uniform distribution over (0,1), prove that $\{X_n: n \geq 0\}$ is a martingale, where $X_n = 2^n \prod_{i=1}^{n} U_i$. (6+4)