Registration Number:

Date & Session:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU -27
M.Sc STATISTICS – III SEMESTER
SEMESTER EXAMINATION: OCTOBER 2023
(Examination conducted in December 2023)
STDE 9420: OPERATIONS RESEARCH

Time: 2 hours **Max Marks: 50**

I. Answer any 5 questions. Each carries ten marks. 5x10=50

1.a) If $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_n)$ is an extreme point of feasible region of linear programming problem(LPP) with m equations and n variables then show that the vector associated with the positive components of $\tilde{x}$ are linearly independent and at most m components of $\tilde{x}$ are positive.

b) Explain the dual simplex algorithm for finding the optimal solution to the given LPP. (6+4)

2. a) Write the dual of the following primal LPP and comments on the dual constraints and decision variables.

Max $Z= 5x_1-2x_2$

subject to the constraints,

$6\ x_1+2x_2=10$

$5x_1-3x_2 \geq 8$

$2x_1-x_2 \leq 5$ $\quad x_2 \geq 0$ and x_1 is unrestricted

b) State and prove sufficient optimality criteria of LPP.

c) If the primal constraint is an equation then show that the associated dual variable is unrestricted. (3+4+3)

3. a) State and prove the Complementary Slackness theorem.

b) Define the following terms:

i. Integer Linear Programming Problem.

ii. Pure Integer Linear Programming Problem

iii. Mixed Integer Linear Programming Problem (7+3)

4. a) What are the applications of integer programming problems?

b) Derive the Gomory constraint in an integer programming problem. (4+6)

5. a) Derive the steady state distribution for a single-channel Poisson arrivals with exponential service (infinite population model).

b) Derive the expected number of units in the system (M/M/I): (∞/FCFS). (7+3)

6) a) Define the Queuing system. Briefly describe the queue discipline and customer's behavior in the queuing theory.

b) If the annual demand for an item is 5,000 units, the ordering cost is Rs 350/- per order, inventory carrying cost is 45% of the purchase price per unit, obtain the optimal order quantity and optimal cost. Find the optimal order size and optimal cost.
The price break is given below:

Quantity	Price (Rs)
$0 < Q < 1200$	12
$1200 \leq Q < 2000$	10
$Q \geq 2000$	9

(5+5)

7. a) Derive the Economic order quantity (EOQ) and total optimal cost for the Inventory model without the shortage.

b) Define the following terms:

i. Demand
ii. Lead time.
iii. Deterministic inventory models. (7+3)

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