Registration Number:
Date & Session:

ST. JOSEPH'S UNIVERSITY, BANGALORE-27
M.Sc.(BIG DATA ANALYTICS)-I SEMESTER
SEMESTER EXAMINATION-OCTOBER 2023
(Examination conducted in November/December 2023)
BDA1321 - LINEAR ALGEBRA AND LINEAR PROGRAMMING
(For current batch students only)

Time: 2 Hours **Max. Marks:** 50

The question paper contains **TWO** printed pages and **THREE** parts

Part A

Answer ALL the questions. **5× 2 = 10**

1. Given $u = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $v = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$. Find $4u$, $(-3v)$, and $4u + (-3)v$.
2. Give a criterion for a non-empty subset of a vector space V to be a subspace of V.
3. Is the function $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (1 + x, 1 + y)$ a linear transformation? Justify your answer.
4. Find the area of the parallelogram spanned by the vectors $u_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $u_2 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ in $\mathbb{R}^2$.
5. Define Linear Programming Problem (LPP) and write its general form.

Part B

Answer any FIVE questions. **5× 4 = 20**

6. Determine if the columns of the matrix $A = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 5 & 8 & 0 \end{bmatrix}$ are linearly independent.
7. Is the union of two subspaces of a vector space a subspace? Justify your answer.
8. For all $u, v \in \mathbb{R}^n$ and $c, d \in \mathbb{R}$, prove the following:
   (a) $c(u + v) = cu + cv$.
   (b) $(c + d)u = cu + du$.

9. Find a solution to the following system :

$$\begin{aligned} 3x_1 + 5x_2 - 4x_3 &= 7 \\ -3x_1 - 2x_2 + 4x_3 &= -1 \\ 6x_1 + x_2 - 8x_3 &= -4. \end{aligned}$$

10. Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is linear, $T(1, 0) = (1, 4)$ and $T(1, 1) = (2, 5)$. Find $T(2, 1)$.

11. Is the following matrix a positive definite? Justify your answer:

$$A = \begin{bmatrix} 1 & -1 & 4 \\ -1 & 2 & 1 \\ 4 & 1 & 16 \end{bmatrix}.$$

12. Show that the set $S = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 + 3x_2 = 5\}$ is convex.

Part C

Answer any TWO questions. **2× 10 = 20**

13. a) For what values of h will y be in $span(\{v_1, v_2, v_3\})$, if $v_1 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}$, $v_3 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$ and $y = \begin{bmatrix} -4 \\ 3 \\ h \end{bmatrix}$. **[5m]**

    b) Let $W = \{(a_1, a_2, a_3, a_4, a_5) \in \mathbb{R}^5 : a_2 + a_3 + a_5 = 0, a_1 = a_4\}$ be a subspace. Find the basis and the dimension of W. **[5m]**

14. a) Diagonalize the following matrix: **[6m]**

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}.$$

    b) For the above matrix A, compute A^{21} using its diagonal form. **[4m]**

15. Solve the following LPP using the graphical method:

$$\text{Maximize } z = 8000x + 7000y$$

subject to

$$\begin{aligned} 3x + y &\leq 66, \\ x + y &\leq 45, \\ x \geq 20, y &\leq 40, \\ x, y &\geq 0. \end{aligned}$$