Register number:

Date and session:

ST JOSEPH'S UNIVERSITY, BENGALURU -27
M.Sc (MATHEMATICS) - II SEMESTER
SEMESTER EXAMINATION: APRIL 2024
(Examination conducted in May/June 2024)
MT 8521- TOPOLOGY

(For current batch students only)

Time 2 Hours | Max Marks: 50

This paper contains TWO printed pages and ONE part.
In question 3 answer either parts a) and b) or answer part c).

I. ANSWER ANY **FIVE** OF THE FOLLOWING.

1. a) Define a closed set. Let $X = \mathbb{R}$ with usual topology. Justify your answer if the subset $A = \{p\}$, singleton set of $\mathbb{R}$ is open or closed. **(4m)**
   b) If X is a set and $\mathcal{B}$ be a Basis for a topology τ on X, then prove that τ equals the collection of all unions of elements from $\mathcal{B}$. **(6m)**
2. a) Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3x + 4$ is a homeomorphism. **(4m)**
   b) If A is a subset of a topological space X and A' is the set of all limit points of A, then prove that $\overline{A} = A \cup A'$. **(6m)**
3. a) State and prove that composition of continuous functions is continuous. **(4m)**
   b) State and prove Pasting lemma. **(6m)**

   OR

   c) Prove that a finite cartesian product of connected spaces is connected. **(10m)**
4. a) Define a connected space. Give any example and justify. **(3m)**
   b) Prove that the image of a connected space under a continuous map is connected. **(7m)**
5. a) If Y is a subspace of X, prove that Y is compact if and only if every covering of Y by sets open in X contains a finite subcollection covering Y. **(6m)**

b) Consider $X = \mathbb{R}$ with the standard topology. Check if the following are a cover for $\mathbb{R}$. Is it an open cover.

i. $A = \{(n, n+2)\}, n \in \mathbb{Z}$.

ii. $B = \{[n, n+2]\}, n \in \mathbb{Z}$.

Give reasons for your answer. **(4m)**

6. State and prove Lebesgue number lemma. **(10m)**

7. a) Prove that the subspace of a first countable space is first countable. **(3m)**

   b) Prove that every metrizable space is normal. **(7m)**