Register Number:
Date:

ST JOSEPH'S UNIVERSITY, BENGALURU-27
OPEN ELECTIVE (MATHEMATICS) - 2^{nd} SEMESTER
SEMESTER EXAMINATION: APRIL 2024
(Examination conducted in May/ June 2024)

MTOE 5: MATHEMATICS FOR PHYSICAL SCIENCES II

(For current batch students only)

Time: 2 Hours **Max Marks:** 60

This question paper contains **TWO** printed pages and **THREE** parts.

PART A

ANSWER ANY SIX FULL QUESTIONS. **(6×2=12)**

1. Find the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^4 + \frac{dy}{dx} = 0$.
2. Find the general solution of the differential equation $\frac{dy}{dx} = y \tan x$.
3. Reduce the differential equation $x\frac{dy}{dx} + y = y^2 \log x$ to linear form with suitable substitution.
4. If $u = x^3 + y^3 + z^3 - 3xyz$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 3u$.
5. Find the stationary points of the function $u = x^3y^2(1 - x - y)$.
6. Find the Laplace transform of $5^t \sin 2t$.
7. Find $\mathcal{L}\left\{\frac{e^{-t}\sin t}{t}\right\}$.
8. Find the inverse Laplace transform of $\frac{s+2}{s^2+36}$.

PART B

ANSWER ANY THREE FULL QUESTIONS. **(3×6=18)**

9. Show that the differential equation $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$ is homogeneous and solve it.
10. Solve $\frac{dy}{dx} + y\cot x = 4x \operatorname{cosec} x$, given that $y = 0$ when $x = \frac{\pi}{2}$.
11. Solve $\frac{dy}{dx} - \left(\frac{1-x}{x}\right)y = -x$.

12. Find $\mathcal{L}\left\{\sin t \sin 3t \sin 5t\right\}$.

13. Find $\mathcal{L}\left\{\cosh t \cdot \sin^3 2t\right\}$.

PART C

ANSWER ANY <u>FIVE FULL</u> QUESTIONS. **(5×6=30)**

14. Solve $(4x+3y+1)\,dx+(3x+2y+1)\,dy=0$.

15. Find the total differential of u and hence find $\dfrac{du}{dt}$ when $u=e^x\sin y$, where $x=\log t,\ y=t^2$.

16. If $x=r\cos\theta$ and $y=r\sin\theta$, find $J=\dfrac{\partial(x,y)}{\partial(r,\theta)}$ and $J'=\dfrac{\partial(r,\theta)}{\partial(x,y)}$ and hence verify $JJ'=1$.

17. Expand $f(x,y)=x^2+xy+y^2$ in powers of $(x-2)$ and $(y-3)$.

18. Find the inverse Laplace transform of the function $\dfrac{s+5}{s^2-6s+13}$.

19. Verify convolution theorem for the functions $f(t)=\sin t,\ g(t)=e^{-t}$.

20. Solve the initial value problem $\dfrac{d^2y}{dt^2}+4\dfrac{dy}{dt}+4y=e^{-t}\ ;\ y(0)=0,\ y'(0)=0$ using Laplace transforms.