ST JOSEPH'S UNIVERSITY, BENGALURU- 27
B.Sc (MATHEMATICS)- 4th SEMESTER
SEMESTER EXAMINATION: APRIL 2024
(Examination conducted in May/June 2024)
MT 422- MATHEMATICS IV
(For current batch students only)

Time: 2 Hours   **Max. Marks:** 60

The paper contains **TWO** printed pages and **THREE** parts.

Part A

Answer any SIX of the following.   [2x6=12]

1. A function $f : G \to G$ defined by $f(x) = 2^x$ is not a homomorphism for a multiplicative group G of non-zero real numbers. Justify.

2. Let $S = \{1, 2, 3, 4\}$, $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$, $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$. Find fog and gof.

3. State Raabe's test.

4. Find the value of a_0 in the Fourier series of $f(x) = x^2$ in $(-\pi, \pi)$.

5. Determine if the function $f(x) = x^3 cosx$ is even or odd. Explain your answer.

6. Find $\dfrac{\Gamma(5)}{\Gamma(3)\Gamma(2)}$

7. Draw the difference table used in Newton's Divided Difference Method for the given set of data $(1, 1), (2, 5), (7, 5), (8, 4)$.

8. Write the Lagrange's Interpolation formula for unequal intervals.

Part B

Answer any THREE of the following.   [3x6=18]

9. Prove that every finite group is isomorphic to a permutation group.

10. Discuss the convergence of the series $\dfrac{2!}{3} + \dfrac{3!}{3^2} + \dfrac{4!}{3^3} + ...$

11. State and prove Cauchy's Root test.

12. The equation $x^6 = x^4 + x^3 + 1$ has one root between 1 and 2. Find this using Newton-Raphson method to three decimals.

Answer any FIVE of the following. [6x5=30]

13. Obtain the Fourier series of the function $f(x) = x$ over the interval $(-\pi, \pi)$ and hence deduce that
$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

14. Obtain the half range Fourier sine series of $f(x) = (x-1)^2$ in the interval $(0,1)$.

15. Prove that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

16. a) Prove that $\Gamma(n+1) = n!$. [3+3]

   b) The velocity (km/min) of an automobile which starts from rest is given in the following table at fixed intervals of time(min)

t	0	2	4	6	8	10	12
v	0	10	18	25	29	32	34

   Find the approximate distance covered by the automobile in 12 minutes using Simpson's one-third rule.

17. Using Newton's forward interpolation formula, find $f(1895)$ for the following data

x	1891	1901	1911	1921	1931
$f(x)$	46	66	81	93	101

18. Calculate the value of $\displaystyle\int_0^1 \frac{dx}{1+x}$ correct upto three decimal places taking six intervals by Trapezoidal rule.

19. Estimate the value of $y(0.1)$ for the initial value problem $\dfrac{dy}{dx} = 1 + x - y$, $y(0) = 1$, $0 \le x \le 1$ using RungeKutta fourth order method with $h = 0.1$.

****************END****************