Register Number:
DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.A. ECONOMICS- I SEMESTER
SEMESTER EXAMINATION: OCTOBER 2018
EC 7418: MATHEMATICAL METHODS FOR ECONOMISTS

Time: 2.5 Hours **Maximum marks 70**

<u>This paper contains two printed page and three parts</u>

PART A Answer any FIVE of the following questions 2X5 = 10

1. Given $S_1 = \{3,6,9\}$, $S_2 = \{a,b\}$, and $S_3 = \{m,n\}$, find the following Cartesian products:
   a. $S_1 \times S_2$
   b. $S_2 \times S_3$
2. Given $A = \begin{pmatrix} 2 & 8 \\ 3 & 0 \\ 5 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 0 \\ 3 & 8 \end{pmatrix}$
   Calculate AB. Can you calculate BA? State the reasons if it is not possible.
3. The demand function is given by, $P = 460 - 3Q$. Find the consumer's surplus when 92 units of the commodities are sold.
4. Check for Walrasian and Marshallian stability given that $q_D = 5 - 2P$ and $q_S = 1 + 3P$.
5. What is a saddle point in game theory?
6. Explain the Hawkins-Simon conditions.
7. What do you mean by basic feasible solution in linear programming?

Part B Answer any THREE of the following: 10 X 3 = 30

8. a) Consider the following national income determination model:
   $Y = C + I + G$; $C = a + b(Y - T)$; $T = d + tY$, $I = I_o$ and $G = G_o$
   where, Y (national income), C (consumption) and T (tax collection) are endogenous variables; I (investment) and G (government expenditure) are exogenous variables; t is the income tax rate. Solve for the endogenous variables, using Cramer's rule.

b) The equilibrium condition for three related markets is given by:

$$11p_1 - p_2 - p_3 = 31$$
$$-p_1 + 6p_2 - 2p_3 = 26$$
$$-p_1 - 2p_2 + 7p_3 = 24$$

Using matrix inversion method, find the equilibrium price for each market.

9. a) The following are the demand functions for two commodities X_1 and X_2:
   $X_1 = p_1^{-1.7}p_2^{0.8}$, $X_2 = p_1^{0.5}p_2^{-0.8}$. Determine whether the two commodities are complements or substitutes.

b) The demand function for a commodity is given by:
$X_1 = 300 - 0.5p_1^2 + 0.02p_2 + 0.05y$. Find the income elasticity of demand when $p_1 = 12$, $p_2 = 10$ and $y = 200$.

10. Given $q = 75[0.3K^{-0.4} + 0.7L^{-0.4}]^{-2.5}$, find out the degree of homogeneity of this production function and verify Euler's theorem.
11. a) If the production function is given by the equation $q = Ax_1^2x_2^2 - Bx_1^3x_2^3$, show that the equation of expansion path is given by $r_1x_1 - r_2x_2 = 0$, where r_1 and r_2 are the unit prices of x1 and x2.

b) A sitar manufacturer can sell x sitars per week at p rupees each, where, $5x = 375 - 3p$. The cost of production is $\{500 + 13x + 1/5\ x^2\}$ rupees. Find how many sitars he should manufacture for maximum profit and what is the profit?

12. The production function of a firm is given by, $q = -3L^3 + 18L^2 + L$. Find the point of inflexion for this function. Show that, at this point marginal productivity (MP) reaches at maximum. Find also the point at which average productivity (AP) reaches at maximum and show that at this point AP=MP.

Part C. Answer any TWO of the following: 15 X 2 = 30

13. Assume that in a duopoly market, the demand and cost functions of the duopolists (Firm A and B) are:

    $P = 100 - 0.5(X_1 + X_2)$, $C_1 = 5X_1,$ $C_2 = 0.5X_2^2$

    a) If firm A is acting as a leader and firm B as a follower, what would be profit maximising quantities for both the firms and also what is the amount of profit for both the firms in this situation?
    b) If firm B is now acting as a leader and firm A as a follower, what would be profit maximising quantities for both the firms and also what is the amount of profit for both the firms in this situation?
    c) Compare the profit maximising quantities and also the amount of profit in the scenarios, a) and b).

14. a) An economy produces only coal and steel. The two commodities serve as intermediate inputs in each other's production. 0.4 tonne of steel and 0.7 tonne of coal are needed to produce a tonne of steel. Similarly, 0.1 tonne of steel and 0.6 tonne of coal are required to produce a tonne of coal. No capital inputs are needed. Do you think that the system is viable? Also, 2 and 5 labour days are required to produce a tonne of coal and steel respectively. If the economy needs 100 tonnes of coal and 50 tonnes of steel, calculate the gross output of the two commodities and total labour required. Also determine the equilibrium prices and the value-added, if the wage rate is Rs. 10 per man day.

    b) Determine the optimum strategies for the two players X and Y and find the value of the game from the following payoff matrix:

$$\begin{array}{cc} & \text{Player Y} \\ \text{Player X} & \begin{pmatrix} 3 & -1 & 4 & 2 \\ -1 & -3 & -7 & 0 \\ 4 & -6 & 2 & -9 \end{pmatrix} \end{array}$$

15. A manufacturer produces two types of medicines, A and B. The profit per bottle of A and B being Rs. 7 and Rs. 5 respectively. Both A and B require two chemicals C and D. Each bottle of A requires 1 litre of C and 4 litres of D whereas each bottle of B requires 2 litres of C and 3 litres of D. The total supply of C and D are 6 litres and 12 litres respectively. Using Simplex method, find, how many bottles of A and B will the firm produce to maximise its profit?