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# ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.Sc. PHYSICS – II SEMESTER

SEMESTER EXAMINATION – APRIL 2019

## PH 8118 : ELECTRODYNAMICS

**Time: 2.5 hours** **Maximum Marks:70**

This question paper contains 2 parts and 3 printed pages.

Some useful Identities:

$$\vec{\nabla}.(f\vec{A})=f(\vec{\nabla}.\vec{A})=\vec{A}.(\vec{\nabla}f)$$

$$\vec{\nabla}.\frac{\hat{r}}{r^2}=4\pi\delta^3(\vec{r})\,where\int\delta^3(r)\,d\tau=1$$

$$\vec{\nabla}x(\vec{A}x\vec{B})=(\vec{B}.\vec{\nabla})\vec{A}-(\vec{A}.\vec{\nabla})\vec{B}+\vec{A}(\vec{\nabla}.\vec{B})-\vec{B}(\vec{\nabla}.\vec{A})$$

### Part-A

**Answer any 5 questions. Each question carries 10 marks.** **(10x5=50)**

1. a) If you release a charge in space then how will it move with respect to equipotential surface? Explain.
   b) A point charge 'q' is situated at a distance d from the center of a grounded conducting sphere of radius a where d>a. For evaluating the potential at a point outside the sphere, find the magnitude and position of image charge using method of images. (2+8)
2. In general the electric potential of an arbitrary localized point charge is given as : $V(\vec{r})=1/(4\pi\epsilon_o)\int(\frac{\rho}{\gamma})(d\tau')$ where $d\tau'$ is the elemental volume of this localized charge distribution whose distance from the origin is $\vec{r}\,'$ and distance from the distribution to the far-off point where potential is being determined is $\vec{\gamma}$ . Now develop a multipole expansion for the precise potential of this charge distribution at a far off point from the source.
3. a) Write down Maxwell's equations. Explain the significance of each equation.
   b) Explain how these equations change in material medium. (6+4)
4. The work necessary to assemble a static charge distribution is given as $\frac{\epsilon_o}{2}\int E^2 d\tau$ where $\vec{E}$ is the resulting electric field and the work required to get the currents going against back emf is $\frac{1}{2\mu_o}\int B^2 d\tau$ where $\vec{B}$ is the resulting magnetic field. If in an instant of time 'dt', the charges move a bit and this configuration of charges and currents changes then calculate the work done on the charges by the electromagnetic force and derive Poynting theorem.

5. The electric and magnetic fields that propagate as plane waves in conductors are of the form $\vec{E}(z,t)=\tilde{E}_o e^{i(\tilde{k}z-\omega t)}$ and $\vec{B}(z,t)=\tilde{B}_o e^{i(\tilde{k}z-\omega t)}$ where propagation vector $\tilde{k}$ is a complex quantity. Here $\tilde{k}=k+i\kappa$ and the real and imaginary parts are given as $k=\omega\sqrt{\frac{\epsilon\mu}{2}}[\sqrt{[1+(\frac{\sigma}{\epsilon\omega})^2]}+1]^{(1/2)}$ and $\kappa=\omega\sqrt{\frac{\epsilon\mu}{2}}[\sqrt{[1+(\frac{\sigma}{\epsilon\omega})^2]}-1]^{(1/2)}$ .
Write the field equations in terms of k and $\kappa$ and interpret the terms. Find the phase relation between these fields. Now, calculate the time averaged energy density of a plane E.M. Wave in a conducting medium and show that it is equal to $\frac{k^2}{2\mu\omega^2}E_o^2 e^{-2\kappa z}$ .

6. The potential formulation of Maxwell's equations obey these wave equations under Lorentz gauge $\nabla^2 V-\mu_o\epsilon_o\frac{\partial^2 V}{\partial t^2}=\frac{-\rho}{\epsilon_o}$ and $\nabla^2 \boldsymbol{A}-\mu_o\epsilon_o\frac{\partial^2 \boldsymbol{A}}{\partial t^2}=-\mu_o \boldsymbol{J}$ . These equations reduce to Poisson's equation and Ampere's law under static case. Write the solutions of these equations i.e. equations for V,**A** under static case. Now using the concept of retarded potentials write these potentials for non-static case. Assuming that the vector potential transforms in the same way as scalar potential, verify the potential relations by showing that the scalar potential in this form satisfies above inhomogenous wave equation.

7. a) Find the scalar product $p^{\mu} p_{\mu}$ and explain what the result signifies.
b) Explain Minkowski's Space-time diagram. (5+5)

## Part-B

**Answer any 4 questions. Each question carries 5 marks. (4x5=20)**

8. The vector potential '**A**' of a magnetic dipole is given as
$\vec{A}_{dip}(r)=(\mu_o/4\pi)(\vec{m}\times\hat{r})/r^2$ where '**m**' is magnetic dipole moment. Show that the magnetic field of this dipole can be written as: $\vec{B}(r)=(\mu_o/4\pi r^3)[3(\vec{m}.\hat{r})\hat{r}-\vec{m}]$ .

9. Suppose an x-y plane forms the boundary between two linear media. An incoming monochromatic plane wave of frequency 'ω', travelling in z-direction, polarized in the plane of incidence (x-z plane) meets the boundary at an angle $\theta_I$. It gives rise to reflected wave at angle $\theta_R$ and transmitted wave at angle $\theta_T$ where $\theta_T < \theta_I$ as velocity of wave in medium 2 $v_2$ is less than than in medium 1 $v_1$ . Assume that all the three laws of geometrical optics are obeyed. The Fresnel's equations for this polarization state are $\tilde{E_{0R}}=(\frac{\alpha-\beta}{\alpha+\beta})\tilde{E_{0I}}$ and $\tilde{E_{0T}}=(\frac{2}{\alpha+\beta})\tilde{E_{0I}}$ where $\tilde{E_{0I}},\tilde{E_{0R}},\tilde{E_{0T}}$ are the incident,reflected and transmitted amplitudes. $\alpha=\frac{\cos\theta_T}{\cos\theta_I}$ And $\beta=\frac{\mu_1 v_1}{\mu_2 v_2}$ .Assuming $\mu_1\approx\mu_2\approx\mu_0$ calculate the transmitted and reflected amplitudes as a function of $\theta_I$ for the air/glass interface at normal incidence, Brewster's angle ,cross over angle and grazing incidence. The index of refraction of glass is 1.5 and air is 1.

10. An atomic clock is placed in a jet plane. The clock measures a time interval of 3600 s when the jet moves with speed 400 m/s. How much larger a time interval does an identical clock held by an observer at rest on the ground measure?

11. Is V ' = V + ax , $\vec{A}'=\vec{A}-at\,\hat{i}$ transformation of ( V , **A** → V ', **A** ') (where V is the electrostatic potential and **A** is the vector potential ) a valid gauge transformation?

12. The electric and magnetic fields in the charge free region z > 0 are given by

$$\vec{E}(\vec{r},t)=E_o e^{-k_1 z}\cos(k_2 x-\omega t)\hat{j}$$

$$\vec{B}(\vec{r},t)=\frac{E_o}{\omega}e^{-k_1 z}k_1\sin(k_2 x-\omega t)\hat{i}+k_2\cos(k_2 x-\omega t)\hat{j}$$

where $\omega$ , $k_1$ and $k_2$ are positive constants. What is the average energy flow ?

13. A neutral pion of rest mass 'm' and relativistic momentum $p=\frac{3}{4}mc$ decays into two photons. One of the photons is emitted in the same direction as the original pion and the other in the opposite direction. Find the relativistic energy of each photon.