Register Number: Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

B.Sc. MATHEMATICS – VI SEMESTER

SEMESTER EXAMINATION: APRIL 2019

MT 6215 -MATHEMATICS

Time: 2 ½ hrs **Max Marks: 70**

This paper contains THREE printed pages and THREE parts.

I Answer any FIVE of the following. **(5 X 2 = 10)**

1. Evaluate : $\lim\limits_{z \to e^{\frac{i\pi}{2}}} \dfrac{z^{10}+1}{z^{8}-1}$.

2. Verify Cauchy-Riemann equations for the function $f(z)=e^{iz}$.

3. Find the fixed points of the bilinear transformation $w=\dfrac{2(z+i)}{z+2i}$.

4. Evaluate $\int_{0}^{2+i} (\bar{z})^2 \, dz$ along the line $2y=x$.

5. Evaluate $\int_C \dfrac{\cos z}{z^2+2z+2}dz$, where C is the circle $|z|=1$.

6. Find $L[t\,5^t+t^5]$.

7. Find the inverse Laplace transform of $\dfrac{s-1}{(s+1)^2}$.

8. If $L[f(t)]=F(s)$, then prove that $L[t\,f(t)]=-F(s)$.

II Answer any SEVEN of the following. **(7 x 6 = 42)**

9. (a) Show that $\arg\left(\frac{z-1+i}{z+i}\right)=\frac{\pi}{4}$ represents a circle. **[3]**

(b) Show that $f(z)=\begin{cases}\frac{xy^2(x+iy)}{x^2+y^4}, & when\ \ z\neq 0\\ 0, & when\ \ z=0\end{cases}$ is not differentiable at $z=0$. **[3]**

10. State and prove the sufficient conditions for a function $f(z)=u+iv$ to be analytic in a domain D.

11. Find the analytic function $f(z)=u+iv$, if $u-v=\frac{\sin x+\cos x-e^{-y}}{2(\cos x-\cosh y)}$.

12. Show that $e^{-x}(x\cos y+y\cos x)$ is harmonic and find it's harmonic conjugate.

13. (a) If $f(z)=u(x,y)+i\,v(x,y)$ is an analytic function, then show that the curves $u(x,y)=c_1$ and $v(x,y)=c_2$ cut orthogonally. **[3]**

(b) If $f(z)$ is an analytic function, show that $\left(\frac{\partial|f(z)|}{\partial x}\right)^2+\left(\frac{\partial|f(z)|}{\partial y}\right)^2=|f'(z)|^2$. **[3]**

14. Show that $\int_C \frac{e^{tz}}{(z^2+1)(z+\sqrt{3})}dz=i\pi\sin\left(t-\frac{\pi}{6}\right)$, where $t>0$ and C is the circle $|z|=\sqrt{2}$.

15. State and prove Cauchy's inequality for a function $f(z)$ of a complex variable z.

16. (a) Discuss the transformation inversion $w=\frac{A}{z}$, where A is a non-zero real constant. **[2]**

(b) Show that the transfomation $w=\frac{1}{z}$ transforms a circle into a circle or a straight line. **[4]**

17. Define "Bilinear transformation". Find the bilinear transformation $w=f(z)$ which maps $\{-1, 1, \infty\}$ in the z-plane onto $\{-i,-1, i\}$ in the W-plane.

III Answer any THREE of the following. **(3 x 6 = 18)**

18. Solve by Gauss-Seidel method:

$$x - 10y + z + 16 = 0$$
$$2x + y + 5z = 13$$
$$10x + y - z = 31$$

19. Use modified Euler's method to compute y for $x = 0.05 \ \ and \ \ 0.1$ given that

$\frac{dy}{dx} = x + y$, with the initial condition $x_0 = 0, \ y_0 = 1$.

20. If $f(t) = \begin{cases} 1 & , 0 < t < \frac{a}{2} \\ -1 & , \frac{a}{2} < t < a \end{cases}$ is a square wave function of period 'a', show that

$L[f(t)] = \frac{1}{s} \tanh\left(\frac{as}{4}\right)$.

21. Using convolution theorem find $L^{-1}\left[\frac{s}{(s^2 + a^2)^2}\right]$.
