Register Number:

DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
B.Sc. MATHEMATICS - IV SEMESTER
SEMESTER EXAMINATION: APRIL 2019
MT-415 MATHEMATICS IV

Time- 1 ½ hrs. **Max Marks-35**

This question paper has ONE printed page

ANSWER ANY SEVEN QUESTIONS. **(7x5=35)**

1. Prove that a subgroup H of a group G is normal if and only if every right coset of H in G is a left coset of H in G.
2. Prove that $f:G\to G$ be a homomorphism from the group G in to itself and H is a cyclic subgroup of G, then $f(H)$ is again a cyclic subgroup of G.
3. State and prove Fundamental theorem of Homomorphism.
4. State and prove Cayley's theorem.
5. Obtain the Fourier series expansion of the function $f(x)=|x| \text{ in } (-\pi,\pi)$ and hence deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots\ldots$.
6. Obtain the Fourier series expansion of the function $f(x)=\begin{cases}2-x & in \quad 0\le x\le 4\\ x-6 & in \quad 4\le x\le 8\end{cases}$
7. Obtain the half range sine series for the function $f(x)=\begin{cases}x & in \quad 0<x<\pi/2\\ \pi-x & in \quad \pi/2<x<\pi\end{cases}$
8. Obtain Taylor's expansion of $\tan^{-1}\left(\frac{y}{x}\right)$ about $(1,1)$ up to second degree term.
9. Test the maximum and minimum of the function $f(x,y)=2x^2-xy+y^2+7x$.
10. Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.