**ST.JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE- 27**

B.Sc. MATHEMATICS –VI SEMESTER

SEMESTER EXAMINATION- APRIL 2018

**MT 6115: MATHEMATICS-VII**

**Time : 2 ½ hrs** **Maximum marks : 70**

*This question paper has three parts and two printed pages .*

**I. Answer any five questions.** **5 x 2 = 10**

1. Determine whether the subset $W = \{ (x_1, x_2, x_3) | x_1^2 + x_2^2 + x_3^2 \leq 0\}$ of $V_3$ (R) is a subspace of $V_3$ (R )?
2. Verify if the vectors (3, 1, 1), (2, – 1, 5) and (4, 0, 3) are linearly independent in $R^3$.
3. Give an example of a linear map which is onto but singular.
4. Form the partial differential equation from $z = a x y + b$
5. Solve the equation $p = e^q$
6. Verify the condition for integrability of the equation (yz + 2x) dx + (zx – 2z) dy + (xy – 2y) dz = 0 .
7. Solve $\frac{dx}{y^2 z} = \frac{dy}{x^2 z} = \frac{dz}{y^2 x}$ .
8. Find $h_1$, $h_2$, $h_3$ for cylindrical coordinates.

**II. Answer any three questions.** **3 x 6 = 18**

9. Prove that in an n dimensional vector space *V(F)*
   (i) any n + 1 elements of V are linearly dependent.
   (ii) no set of n – 1 elements can span V.

10. If T: $R^2 \to R^2$ is a linear transformation such that T (1, 0) = (1, 1) and T (0, 1) = (–1, 2), then show that T maps the square with vertices (0, 0), (1, 0), (1, 1) and (0, 1) into a parallelogram.

11. Find the linear transformation T: $R^3 \rightarrow R^3$ whose range space is spanned by (1 ,0,– 1) and (1, 2, 2)
12. Prove that every vector space $V$ over the real field R of dimension n is isomorphic to $V_n(R)$.

## III. Answer any seven questions. 7 x 6 = 42

13. Solve $\dfrac{dx}{x^2+y^2+yz} = \dfrac{dy}{x^2+y^2-zx} = \dfrac{dz}{z(x+y)}$
14. Form the partial differential equation of all the spheres of radius 3units having their centre on the xy-plane.
15. Solve the partial differential equation $p^3 + q^3 = 27z$
16. Solve $z^2 (p^2 x^2 + q^2) = 1$
17. Solve the given equation using Charpit's method of solution: $z^2 = pqxy$
18. Solve : $2r - s - 3t = 5 e^{x-y}$
19. The ends $A$ and $B$ of a rod $20cm$ long have the temperature at $30^0 C$ and $80^0 C$ until steady state prevails. If the temperatures at $A$ and $B$ are suddenly reduces to $0^0 C$ and maintained $0^0 C$, find the temperature at a distance $x$ from $A$ at time $t$.
20. Express the vector $\vec{f} = 2x\hat{i} - 2y^2\hat{j} + xz\hat{k}$ in cylindrical coordinates.
21. Derive the unit vectors $\hat{e}_\rho, \hat{e}_\theta, \hat{e}_\phi$ in terms of $\hat{i}, \hat{j}, \hat{k}$ for spherical coordinates.