

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
MATHEMATICS – VI SEMESTER
SEMESTER EXAMINATION: APRIL 2018
MT 6216 : MATHEMATICS VIII

Time : $2\frac{1}{2}$ hrs

Maximum marks : 70

This question paper has TWO printed pages and THREE parts.

I Answer any FIVE questions:

(2x5=10)

1. Find the locus of the point z satisfying the relation $|z+i| \leq 3$
2. Evaluate $\lim_{z \rightarrow i} \frac{z^2+1}{z^6+1}$
3. Check whether $u = e^x \cos y + xy$ is harmonic.
4. Evaluate $\oint_C (\bar{z})^2 dz$ around the circle $C: |z|=1$
5. Find the fixed points of the transformation $w = \frac{z-1}{z+1}$
6. Find a real root of $x^3 - 3x + 1.06 = 0$, lying between 0 and 1, using bisection method in two stages, if it exists.
7. Find the Laplace transform of $\sin(mt)$ and $\cos(mt)$.
8. Find the inverse Laplace transform of $\frac{1}{s^2 - 4s + 6}$

II Answer any SEVEN questions:

(6x7=42)

9. Show that $\arg\left(\frac{z-1+i}{z+i}\right) = \frac{\pi}{4}$ represents a circle. Find its centre and radius.
10. Show that $f(z) = \log(z)$ is analytic and hence find $f'(z)$.
11. If $f(z) = u + iv$ is analytic then show that $\left[\frac{\partial}{\partial x}|f(z)|\right]^2 + \left[\frac{\partial}{\partial y}|f(z)|\right]^2 = |f'(z)|^2$
12. Find the analytic function whose imaginary part is $e^x \sin y$.
13. State and prove Cauchy's Integral Theorem.
14. Evaluate $\int_C \frac{z+4}{z^2+2z+5} dz$, where C is the circle $|z+1+i|=2$

15. Find the orthogonal trajectories of the family of curves $x^3y - xy^3 = c$
16. Show that the transformation $w = \frac{1}{z}$ transforms a circle into a circle or to a straight line.
17. Find the bilinear transformation which maps the points $1, i, -1$ onto the points $0, i, \infty$.

III Answer any THREE questions:

(6x3=18)

18. Find the root of the equation $\tan x = x$ near $x=4.5$, correct to four decimal places using Newton-Raphson method.
19. Solve $\frac{dy}{dx} = x + y^2$ with initial condition $y=1$ when $x=0$ for $x=0(0.2)0.4$, using Runge-Kutta method of fourth order.
20. Find the Laplace transform of the function $f(t)$ with period $\frac{2\pi}{w}$ where

$$f(t) = \begin{cases} \sin wt & 0 \leq t \leq \frac{\pi}{w} \\ 0 & \frac{\pi}{w} \leq t \leq \frac{2\pi}{w} \end{cases}$$

21. Using convolution theorem find $L^{-1} \left[\frac{1}{(s+5)(s+3)} \right]$
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