Register Number:
Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
M.Sc. MATHEMATICS - IV SEMESTER
SEMESTER EXAMINATION: APRIL 2018
<u>MT-0416 – THEORY OF NUMBERS</u>

Time- 2 ½ hrs. **Max Marks-70**

This paper contains 2 printed pages.

Answer any <u>seven</u> questions. **(7x10=70)**

1. Verify these for Euler's totient function,
   a) $\phi(p^{\alpha}) = p^{\alpha} - p^{\alpha-1}$ for prime p and $\alpha \geq 1$.
   b) $\phi(mn) = \phi(m)\phi(n)\left(\dfrac{d}{\phi(d)}\right)$, where $d = (m,n)$.
   c) $\phi(mn) = \phi(m)\phi(n)$ if $(m,n) = 1$.
   d) $a \mid b$ implies $\phi(a) \mid \phi(b)$.
   e) $\phi(n)$ is even for $n \geq 3$. Moreover, if n has r distinct odd prime factors, then $2^r \mid \phi(n)$.
   (10)
2. If $n \geq 1$, then prove the following,
   a) $\log n = \sum_{d|n} \Lambda(d)$
   b) $\sum_{d|n} \lambda(d) = \begin{cases} 1, if\ n\ is\ a\ square \\ 0, otherwise \end{cases}$ also $\lambda^{-1}(n) = |\mu(n)|$ for all n. (5+5)
3. State and prove Lagrange's Theorem. (10)
4. Solve for x in $x \equiv 1(\text{mod}3), x \equiv 3(\text{mod}5), x \equiv 6(\text{mod}7), x \equiv 8(\text{mod}11)$ (10)
5. a) State and prove Euler's criterion.
   b) Legendre's symbol is a completely multiplicative function. (7+3)
6. State and prove Gauss's lemma. (10)

7. Let p be an odd prime and let d be any positive divisors of $p-1$. Then in every reduced residue system $\bmod p$ there are exactly $\phi(d)$ numbers a such that $\exp_p(a)=d$. In particular, when $d=\phi(p)=p-1$ there are exactly $\phi(p-1)$ primitive roots $\bmod p$ (10)

8. State and prove Euler's pentagon-number theorem. (10)

9. Determine the upper bound for $p(n)$. (10)

10. State and prove Jacobi's triple product identity. (10)