Register Number:

DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
M.Sc. MATHEMATICS– II SEMESTER
SEMESTER EXAMINATION: APRIL 2018
MT 8515 – TOPOLOGY - II

Time- 2 ½ hrs **Max Marks-70**

This paper contains Two printed pages

Answer any SEVEN of the following questions. **(7x10=70)**

1. a) Prove that a space (X, τ) is Compact if and only if every family of closed sets having finite intersection property has a non-empty intersection.
   b) Define Sequential Compact space. Prove that every sequential compact space is countable compact. [6+4]

2. a) Prove that every compact subspace of a Hausdorff space is closed.
   b) State and prove Lebesgue Number Lemma. [5+5]

3. a) Define Lindelof Space. Prove that every second axiom space is a Lindelof space.
   b) Define Separable space. Prove that every second axiom space is a Separable space. [5+5]

4. Prove that Compactness is Product invariant. [10]

5. a) Define a $T_1 - space$. Prove that a discrete space is a $T_1 - space$. Also prove that $T_1 - space$ is hereditary.
   b) A point x in a $T_1 - space$ (X, τ) is a limit point of a subset A of X if and only if every open set containing x contains infinitely many distinct points of A . [3+7]

6. a) Prove that every convergent sequence in a $T_2 - space$ has a unique limit.
   b) Define Regular space. Prove that a space (X, τ) is Regular if and only if given any open set G and $x \in G$, there is an open set G^* such that $x \in G^* \subseteq \bar{G}^* \subseteq G$. [3+7]

7. Prove that a compact Hausdorff space is normal. [10]

8. Define a Completely Normal space. Prove that a space is completely normal if and only if every subspace is normal. [10]

9. Prove that (X, τ) is normal if and only if for every closed set F *of* X and a real valued continuous function $f : F \rightarrow [a,b]$ there exist a continuous extension $f^* : X \rightarrow [a,b]$ *such that* $f^* |_F = f$. [10]

10. State and prove Urysohn Metrization theorem. [10]
