Register Number:

DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
M.Sc. MATHEMATICS– II SEMESTER
SEMESTER EXAMINATION: APRIL 2018
MT 8515 – TOPOLOGY - II

Time- 2 ½ hrs **Max Marks-70**

This paper contains Two printed pages

Answer any SEVEN of the following questions. **(7x10=70)**

1. a) If every countable open cover of (X, τ) has a finite subcover then prove that (X, τ) is Limit point compact.

   b) Prove that every closed subspace of a compact space is compact. [6+4]

2. a) Prove that every compact space is limit point compact.

   b) State and prove Extreme Value theorem. [5+5]

3. a) State the Countability axioms. Prove that every second axiom space is a first axiom space.

   b) Define Lindelof space. Prove that every second axiom space is a Lindelof space. [5+5]

4. a) Prove that the topologies induced by the Euclidean metric d and square metric ρ are the same as the product topology on R^n.

   b) Prove that a Hausdorff space is product invariant. [5+5]

5. a) Define $T_0 - space$. Prove that a space (X, τ) is a $T_0 - space$ if and only if closure of distinct points in (X, τ) are distinct.

   b) Prove that a space (X, τ) is a $T_1 - space$ if and only if all singleton sets in (X, τ) are closed. [6+4]

6. Define $T_3 - space$. Prove that a metric space is a $T_3 - space$. Prove that $T_3 - space$ is topological. [10]

7. a) Define Normal space. Prove that a space (X, τ) is normal if and only if given any open set G and a closed set $F \subseteq G$ there exist an open set G^* such that $F \subseteq G^* \subseteq \overline{G}^* \subseteq G$.

b) Prove that a closed subspace of a normal space is normal. [7+3]

8. Define a Completely normal space. Prove that a space is completely normal if and only if every subspace is normal. [10]

9. Prove that a space (X, τ) is normal if and only if given any two disjoint closed sets $F_1 \text{ and } F_2 \text{ on } X$ and the interval $[0, 1]$ there exist a continuous function $f : X \to [0,1]$ such that $f(F_1) = \{0\} \text{ and } f(F_2) = \{1\}$. [10]

10. State and prove Urysohn Metrization theorem. [10]
