Register Number:
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ST.JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.Sc. PHYSICS – II SEMESTER

SEMESTER EXAMINATION – APRIL 2018

PH 8115 : ELECTRODYNAMICS

Time: 2.5 hours **Maximum Marks:70**

This question paper contains 2 parts and 3 printed pages.

Some useful Identities:

$$\nabla(\vec{A}.\vec{B})=\vec{A}\,x(\nabla\,x\vec{B})+\vec{B}\,x(\nabla\,x\vec{A})+(\vec{A}.\nabla)\vec{B}+(\vec{B}.\nabla)\vec{A}$$

$$\vec{\nabla}.(\vec{A}\,x\vec{B})=\vec{B}.(\vec{\nabla}\,x\vec{A})-\vec{A}.(\vec{\nabla}\,x\vec{B})$$

$$\vec{\nabla}\,x(\vec{A}\,x\vec{B})=(\vec{B}.\vec{\nabla})\vec{A}-(\vec{A}.\vec{\nabla})\vec{B}+\vec{A}(\vec{\nabla}.\vec{B})-\vec{B}(\vec{\nabla}.\vec{A})$$

In Spherical polar co-ordinates

$$\nabla t=\frac{\partial t}{\partial r}\hat{r}+\frac{1}{r}\frac{\partial t}{\partial\theta}\hat{\theta}+\frac{1}{r\sin\theta}\frac{\partial t}{\partial\varphi}\hat{\varphi}$$

$$\nabla\,x\,\mathbf{v}=\frac{1}{r\sin\theta}[\frac{\partial}{\partial\theta}(\sin\theta\,v_\varphi)-\frac{\partial v_\theta}{\partial\varphi}]\hat{r}+\frac{1}{r}[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\varphi}-\frac{\partial(r\,v_\varphi)}{\partial r}]\hat{\theta}+\frac{1}{r}[\frac{\partial(r\,v_\theta)}{\partial r}-\frac{\partial v_r}{\partial\theta}]\hat{\varphi}$$

All bold letters denote vectors.

Part-A

Answer any 5 questions. Each question carries 10 marks. **(10x5=50)**

1. Using the expression for multipole expansion of vector potential $\vec{A}$ for an arbitrary current loop given by
$\vec{A}(\vec{r})=\frac{\mu_o I}{4\pi}[\frac{1}{r}\oint\vec{dl}'+\frac{1}{r^2}\oint r'\cos\theta'\vec{dl}'+\frac{1}{r^3}\oint(r')^2(3/2\cos^2\theta'-1/2)\vec{dl}'+...]$ where r is the distance from origin to the field point, r' is the distance from origin to the (source) current element on the loop and θ is the angle between r and r'. Show that :
a) Monopoles don't exist.
b) Dipole term can be represented in terms of magnetic dipole moment $\vec{m}$. Knowing that the magnetic moment associated with this current loop of area $\vec{a}$ is given as $\vec{m}=I(\vec{a})$
{hint : Use stoke's theorem to prove $\oint_C T\vec{dl}=-\int_S\vec{\nabla}T\,x\,\vec{da}$ where T is a scalar and use it appropriately to simplify the dipole term in the expansion to derive the required result.}
What does end result signify?
(2+8)

2. a) Faraday concluded from one of his experiments that 'A changing magnetic field produces electric field'. Starting from the flux rule for motional emf and using the definition of emf, arrive at the mathematical formulation of Faraday's statement in differential form(which is also one of the Maxwell's equation).
b) A horizontal metal disc of radius 'a' rotates clockwise with an angular velocity 'ω about the vertical axis, through a uniform magnetic field 'B', pointing up. A circuit is made by connecting one end of the resistor to the axle(or centre) of the disc and the other end to a

sliding contact which touches the outer edge of the disk. Find the current in the resistor and explain its direction of flow.(Is it from centre to outer edge through the resistor or the other way round?).

(5+5)

3. Suppose an x-y plane forms the boundary between two linear media. An incoming monochromatic plane wave of frequency 'ω', polarized perpendicular to the plane of incidence (x-y plane) meets the boundary at an arbitrary angle θ_I. It gives rise to reflected wave at angle θ_R and transmitted wave at angle θ_T where $\theta_T < \theta_I$. Assume that all the three laws of geometrical optics are obeyed. Using appropriate boundary conditions, arrive at the two Fresnel's equations for this polarization state. (10)

4. Reformulate Maxwell's equations in terms of the potentials for the time-dependent configuration and then use Lorentz gauge to simplify them. In what way is this gauge helpful? (10)

5. The retarded potentials of a moving(accelerating) point charge particle at any given time 't' are given as : $V=\frac{1}{4\pi\epsilon_o}\frac{qc}{(rc-\vec{r}.\vec{v})}$ and $\vec{A}=\frac{\vec{v}}{c^2}V(\vec{r},t)$ where V, $\vec{A}$ are scalar and vector potentials respectively, $\vec{v}$ is the velocity of the charge at the retarded time(t_r) and $\vec{r}$ is the vector from the retarded position to the field point. Also the gradient of the scalar potential and time derivative of vector potential are given as:

$$\nabla V=\frac{1}{4\pi\epsilon_o}\frac{qc}{(rc-\vec{r}.\vec{v})^3}[(rc-\vec{r}.\vec{v})\vec{v}-(c^2-v^2+\vec{r}.\vec{a})\vec{r}]$$ and

$$\partial\vec{A}/\partial t=\frac{1}{4\pi\epsilon_o}\frac{qc}{(rc-\vec{r}.\vec{v})^3}[(rc-\vec{r}.\vec{v})(-\vec{v}+\frac{r\vec{a}}{c})+\frac{r}{c}(c^2-v^2+\vec{r}.\vec{a})\vec{v}]$$.

Using the results above, find the associated electric field, magnetic field and poynting vector $\vec{S}$. Which part of $\vec{S}$ represents radiation energy?Explain.{ Use $\vec{u}=c\hat{r}-\vec{v}$ to simplify the field equations.} (6+4)

6. Consider an oscillating dipole made up of two tiny charged metal spheres with charge +q(t) and –q(t) separated by a distance 'd' oscillating with angular frequency ω. The energy radiated by this oscillating electric dipole is given as : $\vec{S}=\frac{\mu_o}{c}[\frac{p_o\omega^2}{4\pi}(\frac{\sin\theta}{r})\cos[\omega(t-r/c)]]^2\hat{r}$. Find the intensity of this radiation and explain the intensity profile. Also, calculate the total power radiated and interpret the result. (10)

7. a) Using the concept of relativistic mass, show that the work-energy theorem holds relativistically.
b) Knowing that force is rate of change of momentum, show how the (ordinary)force transforms as one goes from one inertial frame to another.
Given that the transformation matrix M for transforming from one inertial frame to another is: $M=\begin{pmatrix}\gamma & -\gamma\beta & 0 & 0\\ -\gamma\beta & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$ (5+5)

Part-B

Answer any 4 questions. Each question carries 5 marks. (4x5=20)

8. Find the magnetic field corresponding to the vector potential $\vec{A}=\frac{1}{2}(\vec{F}x\vec{r})+\frac{\vec{r}}{r^3}$ where

$\vec{F}$ is a constant vector. Is it a constant or varying field?

9. A solid sphere of radius R has a charge density ρ given by $\rho=\rho_o(1-\frac{ar}{R})$ whre r is the radial co-ordinate ρ_o, a and R are positive constants. If the magnitude of electric field at $r=R/2$ is 1.25 times that at $r=R$, then find the value of a .

10. The vector potential corresponding to an electromagnetic field is given as $\vec{A}=se^{2t}r\hat{r}$ where 's' is a constant. If this potential under a valid gauge transformation changes to $\vec{A}'=-se^{2t}r\hat{r}$ then what is the corresponding change in scalar potential (i.e. V-V')?

11. A conducting circular loop of radius 'a' has its centre at the origin and its axis makes an angle θ with the z axis. The loop is placed in a magnetic field $\vec{B}=B_o\cos(2\pi f t)\hat{a}_z$ Wb/m^2 .

    a) Derive the expression for the induced voltage on the loop.

    b)If the peak to peak voltage is 16mV at a frequency of 1KHz when the loop lies in the xy plane, determine the loop radius given that B_0=10 x 10^{-3} Wb/m^2.

12. The electric and magnetic fields in a conductor have plane wave solutions but the wave number $\tilde{k}$ is a complex quantity and is given as $\tilde{k}=k+i\kappa$ where

$$k=\omega\sqrt{\frac{\epsilon\mu}{2}}[\sqrt{(1+\frac{\sigma}{\epsilon\omega})}+1]^{(1/2)} \quad \text{and} \quad \kappa=\omega\sqrt{\frac{\epsilon\mu}{2}}[\sqrt{(1+\frac{\sigma}{\epsilon\omega})}-1]^{(1/2)}$$

    Show that the skin depth in good conductor is λ / (2π) where λ is wavelength in conductor. Find the skin depth for a typical metal with $\sigma = 10^7\ (\Omega m)^{-1}$ in the visible range $\omega = 10^{15}$ /s. Assume $\varepsilon = \varepsilon_0$ and $\mu = \mu_0$. Why are metals opaque?

13. Event A happens at point x_A=5, y_A= 0 and z_A= 0 and at time t_A given by ct_A = 3. Event B occurs at x_B=8, y_B = 0 and z_B = 0 and at time t_B given by ct_B= 5, both in system S.
    i) What is the invariant interval between A and B ?
    ii) Is there an inertial frame in which they occur simultaneously? If so, find it's velocity relative to frame S.
    iii) Is there an inertial frame in which they occur at the same point? If so, find it's velocity relative to frame S.