Register Number:
Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
M.Sc. PHYSICS - II SEMESTER
SEMESTER EXAMINATION: APRIL 2018
PH 8415 - QUANTUM MECHANICS I

Time-2 1/2 hrs. **Maximum Marks-70**

This question paper has 4 printed pages and 2 parts

PART A

Answer any FIVE full questions. **MAX. MARKS 5x10=50**

(For some of the questions in Part A, you may need the values of constants listed in Part B).

1.
   (a) (For this question, you may need to use the values of constants given in the opening part of Part B) In order to explore a distance scale of the order of 1 Å what is the energy of photons required? (**2 Marks**)
   (b) Compute the Compton wavelength of the electron. In order to probe the size of the electron, what is the energy of photons that you would require? (**2 Marks**)
   (c) For probing a distance scale of the order of 1×10^{-5} Å using protons
      i. what should be the energy of the protons? (**3 Marks**)
      ii. Will you need to use relativistic effects? Why? (**3 Marks**)

2.
   (a) Let us set up a Young's Double Slit experiment for electrons (see Fig. 1 on page 2). We will arrange the system so that there exists a laser beam along the length of one of the slits (marked "Laser Setup") so that when an electron passes the slit, the photon gets scattered off and does not reach the detector below and sets of a beep in a circuit breaker system (read the caption of Fig. 1 carefully for more explanation).
      i. Sketch (on your answer sheet) the pattern that will be seen on the image screen (not shown in figure, but indicated by the arrow on the right). (**2 Marks**)
      ii. If we switch off the laser beam, what will be the pattern seen on the screen? (**3 Marks**)

   (b) A free particle is described by a wavefunction: $\psi(x,t=0)=A\,e^{-a|x|}$ (which we described in class as one of the forms of a Dirac Delta function).
      i. Normalize this function (express A in terms of a)

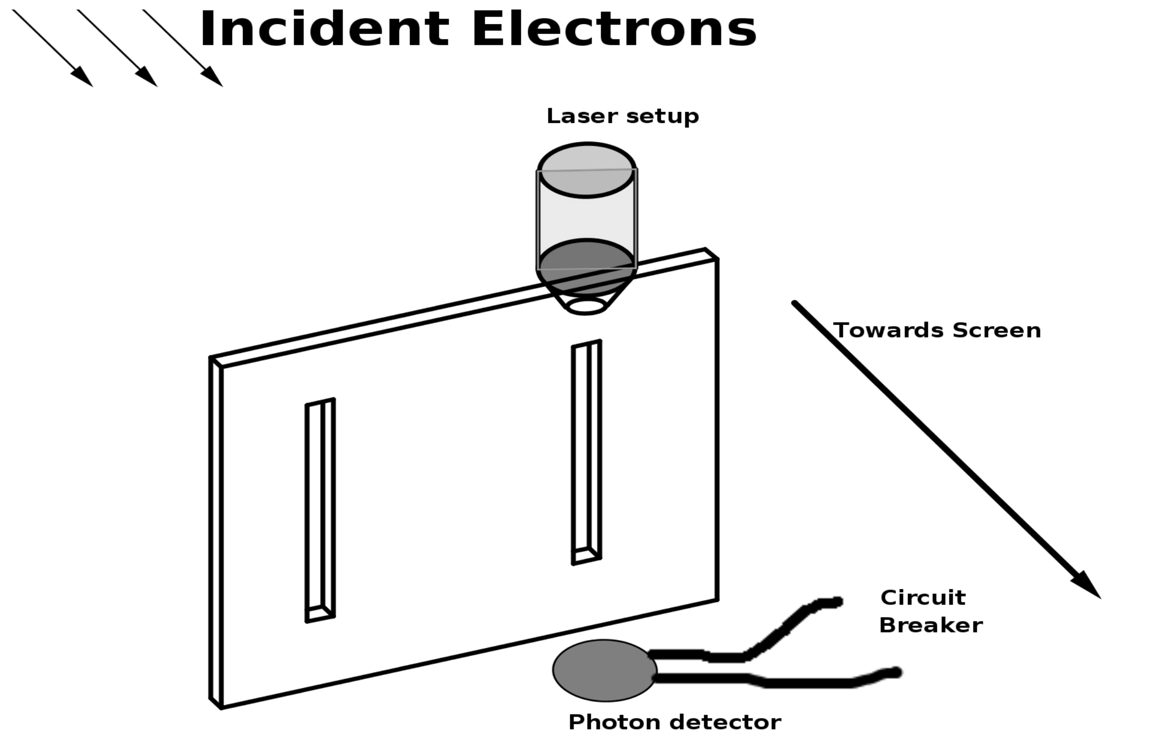

Fig. 1: Double Slit experiment for electrons – the electrons are incident diagonally from top left to bottom right. The "bottle" shaped object is a laser cavity, sending out a laser beam in the vertical direction (downward) and detected by the photon detector, thus setting up a current in the circuit breaker. The laser will need to have a wavelength in X-ray range since it has to "see" the electron. The screen is at a point roughly indicated by the arrow and far away from the slits.

ii. Substitute into the stationary Schrodinger equation and obtain the total energy E of the system having such a wavefunction. (**2 Marks**)

iii. What is the form of the wavefunction after sometime $t=\tau$? (**3 Marks**)

3. Consider an operator $\hat{A}$ that commutes with the Hamiltonian, i.e. $[\hat{A},\hat{H}]$. Show that the expectation value $\langle\hat{A}\rangle$ is a constant for any wavefunction $\psi(x,t)$. The operator does not depend on time explicitly but the wavefunction evolves in time purely due to being a solution of the Schrodinger equation:

$$i\hbar\frac{\partial}{\partial t}\psi(x,t)=-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t)+\hat{V}\psi(x,t)=\hat{H}\psi(x,t)=E\psi(x,t) \quad .$$

4. State and explain with supporting mathematical equations, the postulates of Quantum Mechanics.

5. Consider an operator $\hat{U}_\epsilon(\hat{G})=\mathbf{1}+i\epsilon\hat{G}$ (where ϵ is an infinitesimal parameter and $\hat{G}$ is called the generator of the infinitesimal transformation and $\mathbf{1}$ is the identity matrix – think of the above expansion to be like a Taylor expansion with ϵ with the higher order truncated)

(a) Show that $\hat{U}_\epsilon$ is Unitary if $\hat{G}$ is Hermitian (**1 Mark**)

(b) For a state vector that transforms as $|\psi'\rangle = (\mathbf{1}+\epsilon\hat{G})|\psi\rangle = |\psi\rangle + \delta|\psi\rangle$ what is $\delta|\psi\rangle$ in terms of $\hat{G}$ and ϵ ? (**3 Marks**)

(c) Taking $\hat{S}=\mathbf{1}+i\epsilon\hat{G}$ and $\hat{S}^{\dagger}=\mathbf{1}-i\epsilon\hat{G}$ what would be the transformation of an operator $\hat{A}$ be like (similarity transformation)? (**3 Marks**)

(d) If $\hat{A}$ commutes with $\hat{G}$ what does that mean for the transformation equation you derived in (c)? (**3 Marks**)

6. Starting with the stationary Schrodinger equation in spherical polar coordinates and a potential energy operator $\hat{V}(r)$ that is purely radial, separate out the angular component of the equation. What will the quantum numbers be that label the angular components of the wavefunction? What are the quantum numbers that label the radial component of the wavefunction? (you may need the expression for the Laplacian operator in spherical polar coordinates: $\nabla^2 = \frac{1}{r^2 \sin\theta}\left[\sin\theta\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin\theta}\frac{\partial^2}{\partial\theta^2}\right]$ where the coordinates have their usual meaning)

7. Consider a bead confined to a circular loop of radius R in zero potential (lying in the XY plane), write down the time independent Schrodinger equation.

(a) What should be the boundary conditions on the wavefunction? Sketch the form of the wavefunction along with a sketch of the loop. (**3 Marks**)

(b) What will be the quantized energy of the bead? (you may need the expression for the Laplacian operator in polar coordinates: $\nabla^2=\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}$ where the coordinates have their usual meaning). (**4 Marks**)

(c) What will be the angular momentum of the particle? (**3 Marks**)

PART B

Answer any FOUR full questions. **MAX. MARKS 4x5=20**

[Constants: h=6.6x10^{-34} J s (Planck's constant), 1eV = 1.6x10^{-19} J (electron volt to Joules), c=2.99x10^{8} m/s (speed of light),1Å = 1x10^{-10}m (Angstrom to meters), e = 1.6x10^{-19} C (electronic charge), m_{proton}=1.673x10^{-27}kg (mass of proton), $m_{electron}$=9.109x10^{-31}kg (mass of electron), a = 5.029x10^{-10}m (Bohr radius)]

8. A system is in the state: $|\psi\rangle = \frac{1}{\sqrt{10}}\left(\sqrt{3}|u_1\rangle+\sqrt{2}|u_2\rangle+|u_3\rangle+2|u_4\rangle\right)$ where $|u_n\rangle$ with $n=1,2,3,4$ are the first four eigenstates of the Hamiltonian of the system with energies specified by the eigenvalue equation: $\hat{H}|u_n\rangle = n^2\epsilon|u_n\rangle$

(a) If an experiment is performed to measure energy of the system, what is the probability of getting 9ϵ ? (**2 Marks**)

(b) Let us say that there is another operator $\hat{L}$ that also has $|u_n\rangle$ as its eigenfunctions such that $\hat{L}|u_n\rangle=(2n+1)l_0|u_n\rangle$. If we perform a measurement of energy and obtain 4ϵ and immediately after that we perform an experiment for measuring $\hat{L}$, what is the value that we will obtain? (**3 Marks**)

9. The work function of zinc is $3.74\,\mathrm{eV}$.
   (a) What is the wavelength of photons that can eject electrons with kinetic energy of $100\,\mathrm{eV}$. (**2 Marks**)
   (b) What is the cutoff wavelength for zinc? (**3 Marks**)

10. Find the minimum energy for a photon to convert to an electron-positron pair. What would the photon's frequency and wavelength be?

11. Consider a system that is initially in the state: $|\psi(t=0)\rangle=\frac{1}{5}\begin{pmatrix}3\\4\end{pmatrix}$. The Hamiltonian describing the system is given by $\hat{H}=\begin{pmatrix}2 & 5\\5 & 2\end{pmatrix}$. Find the state of the system after 2 sec.

12. The Ehrenfest theorem says that $\frac{d}{dt}\langle\hat{P}\rangle=\frac{1}{i\hbar}\langle[\hat{P},\hat{V}(x,t)]\rangle$ and $\frac{d}{dt}\langle\hat{X}\rangle=\frac{\langle\hat{P}\rangle}{2m}$. For a **free** particle whose initial position and momentum are given as x_0 and p_0 , show that $\langle\hat{X}\rangle(t)=\frac{p_0 t^2}{m}+x_0$.

13. The operator $\hat{L}^2$ when operating on a state $|l,m\rangle$ gives the following eigenvalue equation: $\hat{L}^2|l,m\rangle=\hbar^2 l(l+1)|l,m\rangle$. For a state having $l=1$, m can take values -1 , 0 or 1 . Write down the eigenvalue equations of $\hat{L}^2$ for all the states.