Register Number:
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ST.JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.Sc. PHYSICS – II SEMESTER

SEMESTER EXAMINATION – APRIL 2017

PH 8115 : ELECTRODYNAMICS

Time: 2.5 hours **Maximum Marks:70**

This question paper contains 2 parts and 3 printed pages.

Some useful Identities:

$\vec{\nabla}.(\vec{A}\,x\,\vec{B})=\vec{B}.(\vec{\nabla}\,x\,\vec{A})-\vec{A}.(\vec{\nabla}\,x\,\vec{B})$

$\vec{\nabla}\,x\,(\vec{A}\,x\,\vec{B})=(\vec{B}.\vec{\nabla})\vec{A}-(\vec{A}.\vec{\nabla})\vec{B}+\vec{A}(\vec{\nabla}.\vec{B})-\vec{B}(\vec{\nabla}.\vec{A})$

In Spherical polar co-ordinates

$$\nabla t=\frac{\partial t}{\partial r}\hat{r}+\frac{1}{r}\frac{\partial t}{\partial\theta}\hat{\theta}+\frac{1}{r\sin\theta}\frac{\partial t}{\partial\varphi}\hat{\varphi}$$

$$\nabla x\,\mathbf{v}=\frac{1}{r\sin\theta}[\frac{\partial}{\partial\theta}(\sin\theta\, v_{\varphi})-\frac{\partial v_{\theta}}{\partial\varphi}]\hat{r}+\frac{1}{r}[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\varphi}-\frac{\partial(r\,v_{\varphi})}{\partial r}]\hat{\theta}+\frac{1}{r}[\frac{\partial(r\,v_{\theta})}{\partial r}-\frac{\partial v_r}{\partial\theta}]\hat{\varphi}$$

All bold letters denote vectors.

Part-A

Answer any 5 questions. Each question carries 10 marks. (10x5=50)

1. a) Find the approximate potential due to an electric dipole at a point far off from the dipole.
   b) Assume that the decrease in the energy stored in the electromagnetic waves within a given volume is due to i)mechanical work done on the charges and ii) some energy that flows out of the surface bounding the volume containing these charges. Write the mathematical equivalent of this statement. What does it signify? (7+3)

2. Suppose we have a piece of magnetised material with magnetisation **M** . Show that the potential of this magnetised object is same as that produced by a volume current $\vec{J}_b=\nabla\,x\,\vec{M}$ throughout the material plus a surface current $\vec{K}=\vec{M}\,x\,\hat{n}$ on the boundary. Given that the vector potential of a single dipole whose dipole moment is '**m**' is given as : $\vec{A}(\mathbf{r})=\frac{\mu_o}{4\pi}\frac{\vec{m}\,x\,\hat{R}}{R^2}$ where **R** is the distance from source point **r'** to field point **r** . [Also, show that $\int_V(\nabla\,x\,\vec{v})\,d\tau=-\oint_S\vec{v}\,x\,d\vec{a}$ and use appropriately as required in the above proof.] (10)

3. a) Write Maxwell's equations in differential form. Explain what each equation signifies.
   b) Using these equations written above, derive these equations in integral form. (6+4)

4. a) Derive the wave equations for propagation of E.M. waves in conducting medium from

Maxwell's equations in material medium. Interpret all the terms.
b) Assuming $\frac{\epsilon}{\sigma}$ to be very small quantity, show that the free charge term can be assumed to be zero. (6+4)

5. If the scalar and vector potential due to sources ρ and **J** are given as
$V(r,t)=\frac{1}{4\pi\epsilon_o}\int\frac{\rho(\boldsymbol{r}')}{R}d\tau'$ $\quad \boldsymbol{A}(\boldsymbol{r})=\frac{\mu_o}{4\pi}\frac{\int \boldsymbol{J}(\boldsymbol{r}')}{R}d\tau'$ where **R** is the distance from source point **r'** to field point **r** .
a) Comment on how and why these equations change when the electromagnetic news from the source travels to the field point(non-static case).
b) If this changed scalar potential also obeys Lorentz gauge $\nabla^2 V-\mu_o\epsilon_o\frac{\partial^2 V}{\partial t^2}=\frac{-\rho}{\epsilon_o}$ then we can justify that our argument for changing these equations is correct.
Assuming that the same argument holds for the scalar and vector potentials, show that this changed scalar potential obeys Lorentz gauge. (4+6)

6. The electric field of an arbitararily moving charged particle is given as:
$\vec{E}(\boldsymbol{r},t)=\frac{q}{4\pi\epsilon_o}\frac{R}{(\vec{R}.\vec{u})^3}[(c^2-v^2)\vec{u}+\vec{R}x(\vec{u}\,x\,\vec{a})]$ where $\vec{u}=c\hat{R}-\vec{v}$ and
$\vec{B}(\boldsymbol{r},t)=\frac{1}{c}\hat{R}\,x\,\vec{E}(\boldsymbol{r},t)$
where **R** is the distance from retarded position **r '** to field point **r** .Using the expression of fields as given above, derive an expression for the Larmor formula i.e. the total power radiated by this point charge particle. (10)

7. Consider a large parallel plate capacitor of length l_o and width 'w' that carries charges $+\sigma_o$ and $-\sigma_o$ on each plate. If the capacitor is at rest in the reference frame S_o the uniform field between the plates of this capacitor is $E_o=\frac{\sigma_o}{\epsilon_o}\hat{y}$.
i) If the same capacitor is now seen from reference frame S which is moving to the right with speed v_o,explain how the parallel and perpendicular components of this field will transform in frame S.
ii) Now consider another frame $\bar{S}$ which moves with velocity $\bar{v}$ relative to S_o and v relative to S. If the surface current (current per unit width) in the frame S is given as **K**±= $\mp\sigma v_o\hat{x}$. Explain how fields E_y and B_z in the frame S transform in frame $\bar{S}$. (10)

Part-B

Answer any 4 questions. Each question carries 5 marks. (4x5=20)

8. If the electrostatic potential $V(r,\theta,\varphi)$ in a region where the charge density ρ is zero is given as $V(r,\theta,\varphi)=f(r)\cos\theta$ then show that the value of $f(r)=ar+\frac{b}{r^2}$ satisfies the above relation of V for this potential.

9. Consider a solenoid of radius R with n turns per unit length, in which a time dependent slowly varying current $I = I_0 \sin\omega t$ flows. Find the magnitude and direction of the electric field at a perpendicular distance r (both inside and outside) from the axis of symmetry of the solenoid.

10. The electric field of an electromagnetic wave is given by
$\vec{E} = E_o \cos[\pi(0.3x + 0.4y - 1000t)]\hat{k}$. Find the magnetic field.

11. The electric and magnetic fields in the charge free region z > 0 are given by
$\vec{E}(\vec{r},t) = E_o e^{-k_1 z} \cos(k_2 x - \omega t)\hat{j}$
$\vec{B}(\vec{r},t) = \frac{E_o}{\omega} e^{-k_1 z} k_1 \sin(k_2 x - \omega t)\hat{i} + k_2 \cos(k_2 x - \omega t)\hat{j}$

where ω , k $_1$ and k $_2$ are positive constants. What is the average energy flow in the x - direction?

12. Suppose the xz -plane forms a chargeless boundary between two media of permittivities $\in_{\text{left}}$ and $\in_{\text{right}}$ where $\in_{\text{left}} : \in_{\text{right}} = 1 : 2$. If the uniform electric field on the left is

$\vec{E}_{\text{left}} = c(3\hat{i} + 2\hat{j} + 2\hat{k})$ (where c is a constant), then what is the electric field on the right E_{right}?

13. a) Is V ′ = V + ax , $\vec{A}' = \vec{A} - at\,\hat{i}$ transformation of (V , **A** → V ', **A** ') (where V is the electrostatic potential and **A** is the vector potential) a valid gauge transformation?
b) Show that the four- dimensional scalar product is invariant under Lorentz transformation where the transformation matrix M for transforming from reference frame S to $\bar{S}$ is

$$M = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Here $\gamma = \frac{1}{\sqrt{(1 - v^2/c^2)}}$ $\beta = v/c$ and v is the velocity of $\bar{S}$ relative to S. (2+3)