Register Number:
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ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27

M.Sc. PHYSICS - III SEMESTER

SEMESTER EXAMINATION: OCTOBER 2021

(Examination conducted in January-March 2022)

PH 9120 - QUANTUM MECHANICS-II

Time-2 1/2 hrs. Max Marks-70

This question paper has 5 printed pages and 2 parts

Part A

Answer any 5 questions **(5x10=50)**

1. In quantum mechanics, what are the various ways Degeneracies occur? Explain

2. In the classical limit, the de-Broglie wavelength is far less than the size of the system ($\lambda \ll x$). Further, in the classical limit, the total energy of the system is $E=\frac{p^2}{2m}-V(x)$. With the condition that $\frac{\delta\lambda}{\lambda}\sim\frac{d\lambda}{dx}$ show that the validity of WKB approximation implies that $\delta\lambda\sim\left|\frac{mh^2}{p^4}\frac{dV}{dx}\right|\ll 1$.

3.
   (a) Write down (you don't have to derive) the perturbation equations up to the second order for a non-degenerate system described by a Hamiltonian $H^{(0)}$ and perturbed by an energy function $W=\lambda\hat{W}$ where λ is the perturbative scale factor.
   (b) Obtain the first order perturbation to the state function (use the wavefunction form, not the vectorial notation). [2+8]

4. For a two fold degenerate system described by a Hamiltonian $H^{(0)}$ and perturbed by an energy function $W=\lambda\hat{W}$ where λ is the perturbative scale factor, we know that the

equation for the first order perturbative change in energy gives us the following equation: $\begin{vmatrix} \langle \phi_n^{(0)1} | \hat{W} | \phi_n^{(0)1} \rangle - E_n^{(1)} & \langle \phi_n^{(0)1} | \hat{W} | \phi_n^{(0)2} \rangle \\ \langle \phi_n^{(0)2} | \hat{W} | \phi_n^{(0)1} \rangle & \langle \phi_n^{(0)2} | \hat{W} | \phi_n^{(0)2} \rangle - E_n^{(1)} \end{vmatrix} = 0$. What is the value of $E_n^{(1)}$?

5. Assuming that time dependent perturbation induces a transition to one of the stationary states of the unperturbed system: $|\psi(t)\rangle = \sum_n c_n(t) |\phi_n\rangle$, and the system to be described by a Hamiltonian $H^{(0)}$ that is perturbed by an energy function $W(t) = \lambda \hat{W}(t)$ with λ being a scale factor for the perturbation, obtain an evolution equation for the coefficients *after factoring out the time evolution due to the time dependent Schrodinger equation*.

6.
   (a) Derive the Hamiltonian of a system of two particles in a potential dependent only on the distance between them in the center of mass frame.
   (b) Express the Hamiltonian in reduced mass, and explain all terms. [6+4]

7. Describe scattering by a potential with a figure. Explain the various terms and obtain the asymptotic form for the scattered wave.

Part B

Answer any 4 questions **(4x5=20)**

[Constants: **h**=6.626070x10^{-34} J s (**Planck's constant**), **1eV** = 1.6x10^{-19} J (**electron volt to Joules**), **c**=2.99792458x10^{8} m/s (**speed of light**), **1Å** = 1x10^{-10}m (**Angstrom to meters**), **e** = 1.602176x10^{-19} C (**electronic charge**), **ε_0**= 8.85418782x10^{-12}m^{-3}kg^{-1}s^{4}A^{2} (**permittivity of free space**), **m$_{proton}$**=1.672621898x10^{-27}kg (**mass of proton**), **m$_{electron}$**=9.10938356x10^{-31}kg (**mass of electron**), **m$_{neutron}$**=1.674927471x10^{-27}kg (**mass of neutron**), **a** = 5.029x10^{-10}m (**Bohr radius**), **α** = 1/137 (**Fine Structure Constant**), **G**=6.674x10^{-11}m^{3}kg^{-1}s^{-2} (**Gravitational constant**), **M$_\odot$**=1.9891x10^{30} kg (**Solar mass**), **R$_\odot$**=6.9x10^{8} m (**Sun's Radius**), **σ** = 5.67x10^{-8} W m^{-2} K^{-4} (**Stefan-Boltzmann constant**), **M$_{Earth}$**=5.97x10^{27}kg (**Mass of Earth**), **D$_{earth-sun}$**=1.49x10^{11}m (**Earth-Sun distance**), 1 inch = 2.54 cm, 1foot=12 inches]

8. Two mutually non-interacting spin half particles (Fermions) are moving in an infinite (particle in a box) potential. The energy and wavefunction for individual particles in the infinite potential are given as: $E_n = \frac{\hbar^2}{2m} \frac{n^2 \pi^2}{L^2}$ and $\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right)$ respectively. For the composite system, obtain the energy of the first excited state and write down the wavefunction (including the angular component – you can use the Table of Clebsch-Gordan coefficients available in the question paper).

9. The Hamiltonian of a two-level system described by the particle in a box potential of length

, experiences a perturbation $W=10^{-3}E_1^{(0)}\frac{x}{L}$ where $E_1^{(0)}$ is the first unperturbed eigenstate for a particle in a box. Obtain the first order change in the energies of the two levels. The energies and state functions for the unperturbed particle in a box are given as: $E_n^{(0)}=\frac{\hbar^2}{2m}\frac{n^2\pi^2}{L^2}$ and $\psi_n^{(0)}=\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$ respectively.

10. What is probability of transition from the ground state $n=1$ to the first excited state $n=2$ for a perturbation of $W=10^{-3}E_1^{(0)}\sin\omega t$ to a system described by the particle in a box potential?

11. Use the variational method to find the optimal values for b and the corresponding approximate ground state and ground state energy for a simple harmonic oscillator $V=\frac{1}{2}m\omega^2x^2$ where m is the mass of the particle and ω the frequency of the oscillator. Use the trial wavefunction: $\psi_{\text{trial}}(x)=\frac{A}{x^2+b}$. The average kinetic energy for the system works out to be: $\langle T\rangle=\frac{\hbar^2}{4bm}$. You may need to use some of the standard integrals given later in the question paper.

12.
(a) Show that the ground state for a system is always a lower bound to the expectation value of energy for a system.
(b) What are the conditions on the wavefunction of the system for this to happen? [4+1]

13. Using the Born approximation for scattering amplitude: $f_k^{(B)}(\theta,\phi)=\frac{-\mu}{2\pi\hbar^2}\int V(\vec{r})d^3r$ compute the scattering amplitude and differential scattering cross section of scattering by a soft-sphere potential: $V(\vec{r})=\begin{cases}V_0 & if\, r\leq a\\ 0 & if\, r>a\end{cases}$.

Table of (some) Integrals

Gamma Function:
$\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}\,dt$
$\Gamma(n)=(n-1)!$
$\Gamma(\frac{1}{2}+n)=\frac{(2n)!}{4^n n!}\sqrt{\pi}$

(a) $\int_0^\infty e^{-2bt}\,dt=\frac{1}{2b}$

(b) $\int_0^\infty t\,e^{-2bt}\,dt=\frac{1}{4b^2}$

(c) $\int_0^\infty t^2 e^{-2bt}\,dt=\frac{1}{4b^3}$

(d) $\int_0^\infty t^3 e^{-2bt}\,dt=\frac{3}{8b^4}$

(e) $\int_0^\infty t^4 e^{-2bt}\,dt=\frac{3}{4b^5}$

(f) $\int_0^\infty t^5 e^{-2bt}\,dt=\frac{15}{8b^6}$

(g) $\int_0^\infty t^6 e^{-2bt}\,dt=\frac{45}{8b^7}$

(h) $\int \frac{1}{t^2+b^2}\,dt=\frac{1}{b}\tan^{-1}\left(\frac{t}{b}\right)$

(i) $\int \frac{1}{(t^2+b^2)^2}\,dt=\frac{1}{2b^3}\left(\frac{bt}{(b^2+t^2)}+\tan^{-1}\left(\frac{t}{b}\right)\right)$

(j) $\int_0^\infty t^4 e^{-\alpha^2 t^2}\,dt=\frac{3\sqrt{\pi}}{8\alpha^5}$

(k) $\int \frac{1}{(t^2+b^2)^4}\,dt=\frac{1}{16b^7}\left(\frac{15t^5 b+40b^3 5^3+33b^5 t}{(3t^6+9bt^4+9b^3t^2+b^5)}+5\tan^{-1}\left(\frac{t}{b}\right)\right)$

(l) $\int \frac{t^2}{(t^2+b^2)^2}\,dt=\left(-\frac{t}{(2b^2+2t^2)}+\frac{1}{2b}\tan^{-1}\left(\frac{t}{b}\right)\right)$

(m) $\int \frac{1}{(t^2+b^2)^3}\,dt=\frac{3}{8b^5}\left(\frac{5/3\,b^3 t+bt^3}{(b^2+t^2)^2}+\tan^{-1}\left(\frac{t}{b}\right)\right)$

(n) $\int \frac{t^2}{(t^2+b^2)^4}\,dt=\frac{1}{16b^5}\left(\frac{bt^5+8/3\,b^3t^3-b^5t}{(b^2+t^2)^3}+\tan^{-1}\left(\frac{t}{b}\right)\right)$

(o) $\int \frac{t^4}{(t^2+b^2)^4}\,dt=\frac{1}{16b^3}\left(\frac{bt^5+8/3\,b^3t^3-b^5t}{(b^2+t^2)^3}+\tan^{-1}\left(\frac{t}{b}\right)\right)$

(p) $\int \frac{t^6}{(t^2+b^2)^4}\,dt=\frac{1}{16b}\left(\frac{11bt^5+40/3\,b^3t^3-5b^5t}{(b^2+t^2)^3}+5\tan^{-1}\left(\frac{t}{b}\right)\right)$

(q) $\int \sqrt{a/x-1}\,dx=x\sqrt{a/x-1}+a\ \tan^{-1}\left(\sqrt{a/x-1}\right)$

(r) $\int \sqrt{1-ax}\,dx=-\frac{2(1-ax)^{3/2}}{3a}$

(s) $\int \sqrt{1-ax^2}\,dx=\frac{1}{2}x\sqrt{1-ax^2}+\frac{\sin^{-1}\sqrt{a}\,x}{2\sqrt{a}}$

(t) $\int_{-\infty}^{\infty} e^{-\alpha t^2+i\omega t}\,dt=\sqrt{\frac{\pi}{\alpha}}\,e^{-\frac{\omega^2}{4\alpha}}$

(u) $\int_0^\infty t^n e^{-st}\,dt=\frac{n!}{s^{n+1}}$ (Laplace Transform)

34. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for −8/15 read $-\sqrt{8/15}$.

Notation: column headers J, M; rows m_1, m_2; entries: Coefficients.

$$Y_1^0=\sqrt{\frac{3}{4\pi}}\cos\theta \qquad Y_1^1=-\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0=\sqrt{\frac{5}{4\pi}}\Big(\frac{3}{2}\cos^2\theta-\frac{1}{2}\Big) \qquad Y_2^1=-\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi} \qquad Y_2^2=\frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m}=(-1)^m Y_\ell^{m*} \qquad d^\ell_{m,0}=\sqrt{\frac{4\pi}{2\ell+1}}\,Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^j_{m',m}=(-1)^{m-m'}d^j_{m,m'}=d^j_{-m,-m'}$$

1/2×1/2

m_1	m_2	J=1, M=+1
+1/2	+1/2	1

m_1	m_2	J=1, M=0	J=0, M=0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	J=1, M=−1
−1/2	−1/2	1

1×1/2

m_1	m_2	J=3/2, M=+3/2
+1	+1/2	1

m_1	m_2	J=3/2, M=+1/2	J=1/2, M=+1/2
+1	−1/2	1/3	2/3
0	+1/2	2/3	−1/3

m_1	m_2	J=3/2, M=−1/2	J=1/2, M=−1/2
0	−1/2	2/3	1/3
−1	+1/2	1/3	−2/3

m_1	m_2	J=3/2, M=−3/2
−1	−1/2	1

2×1/2

m_1	m_2	J=5/2, M=+5/2
+2	+1/2	1

m_1	m_2	J=5/2, M=+3/2	J=3/2, M=+3/2
+2	−1/2	1/5	4/5
+1	+1/2	4/5	−1/5

m_1	m_2	J=5/2, M=+1/2	J=3/2, M=+1/2
+1	−1/2	2/5	3/5
0	+1/2	3/5	−2/5

m_1	m_2	J=5/2, M=−1/2	J=3/2, M=−1/2
0	−1/2	3/5	2/5
−1	+1/2	2/5	−3/5

m_1	m_2	J=5/2, M=−3/2	J=3/2, M=−3/2
−1	−1/2	4/5	1/5
−2	+1/2	1/5	−4/5

m_1	m_2	J=5/2, M=−5/2
−2	−1/2	1

3/2×1/2

m_1	m_2	J=2, M=+2
+3/2	+1/2	1

m_1	m_2	J=2, M=+1	J=1, M=+1
+3/2	−1/2	1/4	3/4
+1/2	+1/2	3/4	−1/4

m_1	m_2	J=2, M=0	J=1, M=0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	J=2, M=−1	J=1, M=−1
−1/2	−1/2	3/4	1/4
−3/2	+1/2	1/4	−3/4

m_1	m_2	J=2, M=−2
−3/2	−1/2	1

2×1

m_1	m_2	J=3, M=+3
+2	+1	1

m_1	m_2	J=3, M=+2	J=2, M=+2
+2	0	1/3	2/3
+1	+1	2/3	−1/3

m_1	m_2	J=3, M=+1	J=2, M=+1	J=1, M=+1
+2	−1	1/15	1/3	3/5
+1	0	8/15	1/6	−3/10
0	+1	2/5	−1/2	1/10

m_1	m_2	J=3, M=0	J=2, M=0	J=1, M=0
+1	−1	1/5	1/2	3/10
0	0	3/5	0	−2/5
−1	+1	1/5	−1/2	3/10

m_1	m_2	J=3, M=−1	J=2, M=−1	J=1, M=−1
0	−1	2/5	1/2	1/10
−1	0	8/15	−1/6	−3/10
−2	+1	1/15	−1/3	3/5

m_1	m_2	J=3, M=−2	J=2, M=−2
−1	−1	2/3	1/3
−2	0	1/3	−2/3

m_1	m_2	J=3, M=−3
−2	−1	1

1×1

m_1	m_2	J=2, M=+2
+1	+1	1

m_1	m_2	J=2, M=+1	J=1, M=+1
+1	0	1/2	1/2
0	+1	1/2	−1/2

m_1	m_2	J=2, M=0	J=1, M=0	J=0, M=0
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3

m_1	m_2	J=2, M=−1	J=1, M=−1
0	−1	1/2	1/2
−1	0	1/2	−1/2

m_1	m_2	J=2, M=−2
−1	−1	1

3/2×1

m_1	m_2	J=5/2, M=+5/2
+3/2	+1	1

m_1	m_2	J=5/2, M=+3/2	J=3/2, M=+3/2
+3/2	0	2/5	3/5
+1/2	+1	3/5	−2/5

m_1	m_2	J=5/2, M=+1/2	J=3/2, M=+1/2	J=1/2, M=+1/2
+3/2	−1	1/10	2/5	1/2
+1/2	0	3/5	1/15	−1/3
−1/2	+1	3/10	−8/15	1/6

m_1	m_2	J=5/2, M=−1/2	J=3/2, M=−1/2	J=1/2, M=−1/2
+1/2	−1	3/10	8/15	1/6
−1/2	0	3/5	−1/15	−1/3
−3/2	+1	1/10	−2/5	1/2

m_1	m_2	J=5/2, M=−3/2	J=3/2, M=−3/2
−1/2	−1	3/5	2/5
−3/2	0	2/5	−3/5

m_1	m_2	J=5/2, M=−5/2
−3/2	−1	1

3/2×3/2

m_1	m_2	J=3, M=+3
+3/2	+3/2	1

m_1	m_2	J=3, M=+2	J=2, M=+2
+3/2	+1/2	1/2	1/2
+1/2	+3/2	1/2	−1/2

m_1	m_2	J=3, M=+1	J=2, M=+1	J=1, M=+1
+3/2	−1/2	1/5	1/2	3/10
+1/2	+1/2	3/5	0	−2/5
−1/2	+3/2	1/5	−1/2	3/10

m_1	m_2	J=3, M=0	J=2, M=0	J=1, M=0	J=0, M=0
+3/2	−3/2	1/20	1/4	9/20	1/4
+1/2	−1/2	9/20	1/4	−1/20	−1/4
−1/2	+1/2	9/20	−1/4	−1/20	1/4
−3/2	+3/2	1/20	−1/4	9/20	−1/4

m_1	m_2	J=3, M=−1	J=2, M=−1	J=1, M=−1
+1/2	−3/2	1/5	1/2	3/10
−1/2	−1/2	3/5	0	−2/5
−3/2	+1/2	1/5	−1/2	3/10

m_1	m_2	J=3, M=−2	J=2, M=−2
−1/2	−3/2	1/2	1/2
−3/2	−1/2	1/2	−1/2

m_1	m_2	J=3, M=−3
−3/2	−3/2	1

2×3/2

m_1	m_2	J=7/2, M=+7/2
+2	+3/2	1

m_1	m_2	J=7/2, M=+5/2	J=5/2, M=+5/2
+2	+1/2	3/7	4/7
+1	+3/2	4/7	−3/7

m_1	m_2	J=7/2, M=+3/2	J=5/2, M=+3/2	J=3/2, M=+3/2
+2	−1/2	1/7	16/35	2/5
+1	+1/2	4/7	1/35	−2/5
0	+3/2	2/7	−18/35	1/5

m_1	m_2	J=7/2, M=+1/2	J=5/2, M=+1/2	J=3/2, M=+1/2	J=1/2, M=+1/2
+2	−3/2	1/35	6/35	2/5	2/5
+1	−1/2	12/35	5/14	0	−3/10
0	+1/2	18/35	−3/35	−1/5	1/5
−1	+3/2	4/35	−27/70	2/5	−1/10

m_1	m_2	J=7/2, M=−1/2	J=5/2, M=−1/2	J=3/2, M=−1/2	J=1/2, M=−1/2
+1	−3/2	4/35	27/70	2/5	1/10
0	−1/2	18/35	3/35	−1/5	−1/5
−1	+1/2	12/35	−5/14	0	3/10
−2	+3/2	1/35	−6/35	2/5	−2/5

m_1	m_2	J=7/2, M=−3/2	J=5/2, M=−3/2	J=3/2, M=−3/2
0	−3/2	2/7	18/35	1/5
−1	−1/2	4/7	−1/35	−2/5
−2	+1/2	1/7	−16/35	2/5

m_1	m_2	J=7/2, M=−5/2	J=5/2, M=−5/2
−1	−3/2	4/7	3/7
−2	−1/2	3/7	−4/7

m_1	m_2	J=7/2, M=−7/2
−2	−3/2	1

2×2

m_1	m_2	J=4, M=+4
+2	+2	1

m_1	m_2	J=4, M=+3	J=3, M=+3
+2	+1	1/2	1/2
+1	+2	1/2	−1/2

m_1	m_2	J=4, M=+2	J=3, M=+2	J=2, M=+2
+2	0	3/14	1/2	2/7
+1	+1	4/7	0	−3/7
0	+2	3/14	−1/2	2/7

m_1	m_2	J=4, M=+1	J=3, M=+1	J=2, M=+1	J=1, M=+1
+2	−1	1/14	3/10	3/7	1/5
+1	0	3/7	1/5	−1/14	−3/10
0	+1	3/7	−1/5	−1/14	3/10
−1	+2	1/14	−3/10	3/7	−1/5

m_1	m_2	J=4, M=0	J=3, M=0	J=2, M=0	J=1, M=0	J=0, M=0
+2	−2	1/70	1/10	2/7	2/5	1/5
+1	−1	8/35	2/5	1/14	−1/10	−1/5
0	0	18/35	0	−2/7	0	1/5
−1	+1	8/35	−2/5	1/14	1/10	−1/5
−2	+2	1/70	−1/10	2/7	−2/5	1/5

m_1	m_2	J=4, M=−1	J=3, M=−1	J=2, M=−1	J=1, M=−1
+1	−2	1/14	3/10	3/7	1/5
0	−1	3/7	1/5	−1/14	−3/10
−1	0	3/7	−1/5	−1/14	3/10
−2	+1	1/14	−3/10	3/7	−1/5

m_1	m_2	J=4, M=−2	J=3, M=−2	J=2, M=−2
0	−2	3/14	1/2	2/7
−1	−1	4/7	0	−3/7
−2	0	3/14	−1/2	2/7

m_1	m_2	J=4, M=−3	J=3, M=−3
−1	−2	1/2	1/2
−2	−1	1/2	−1/2

m_1	m_2	J=4, M=−4
−2	−2	1

$$d^1_{0,0}=\cos\theta \qquad d^{1/2}_{1/2,1/2}=\cos\frac{\theta}{2} \qquad d^{1/2}_{1/2,-1/2}=-\sin\frac{\theta}{2}$$

$$d^1_{1,1}=\frac{1+\cos\theta}{2} \qquad d^1_{1,0}=-\frac{\sin\theta}{\sqrt{2}} \qquad d^1_{1,-1}=\frac{1-\cos\theta}{2}$$

$$d^{3/2}_{3/2,3/2}=\frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2}=-\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2}=\sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2}=-\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2}=\frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2}=-\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^2_{2,2}=\Big(\frac{1+\cos\theta}{2}\Big)^2 \qquad d^2_{2,1}=-\frac{1+\cos\theta}{2}\sin\theta \qquad d^2_{2,0}=\frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^2_{2,-1}=-\frac{1-\cos\theta}{2}\sin\theta \qquad d^2_{2,-2}=\Big(\frac{1-\cos\theta}{2}\Big)^2$$

$$d^2_{1,1}=\frac{1+\cos\theta}{2}(2\cos\theta-1) \qquad d^2_{1,0}=-\sqrt{\frac{3}{2}}\sin\theta\cos\theta \qquad d^2_{1,-1}=\frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^2_{0,0}=\Big(\frac{3}{2}\cos^2\theta-\frac{1}{2}\Big)$$

Figure 34.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The coefficients here have been calculated using computer programs written independently by Cohen and at LBNL.