Register Number:
Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU- 27
M.Sc MATHEMATICS- I SEMESTER
SEMESTER EXAMINATION: OCTOBER 2021
(Examination conducted in January-March 2022)
MT 7421: Ordinary Differential Equations

Duration: 2.5 Hours **Max. Marks:**70

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1. The paper contains **TWO** printed pages.
2. Answer any **SEVEN FULL** questions, where each question carries 10 marks.

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1. Find the power series solution in powers of $(x-1)$ of the initial value problem $x^2y'' + y' + 2y = 0$ given that $y(1) = 1$ and $y'(1) = 2$.

2. Show that $x = 0$ is the regular singular point and hence obtain the series solution of the given differential equation $2x^2y'' + xy' - (x+1)y = 0$. **[10 marks]**

3. (a) Show that for every integer n, $J_{-n}(x) = (-1)^n J_n(x)$ where $J_n(x)$ is Bessel's function of first kind and of order n. **[7 marks]**
   (b) Prove that $\frac{d}{dx}\{x^n J_n(x)\} = x^n J_{n-1}(x)$. **[3 marks]**

4. Find the eigenvalue and the corresponding eigen function of $\frac{d}{dx}(x\frac{dy}{dx}) + \frac{\lambda}{x}y = 0$ with $y(1) = 0$ and $y'(e^{2\pi}) = 0$. **[10 marks]**

5. (a) Determine if the boundary operators are linearly dependent or not.
   $U_1(u) = u_1 - 3u_3 + u_4$ and $U_2(u) = u_1 + u_3 - 2u_4$ **[3 marks]**
   (b) Solve the system of differential equations
   $\frac{dx_1}{dt} = -4x_1 - x_2 + 9e^{-3t}$ and $\frac{dx_2}{dt} = x_1 + x_2 - 5e^{-3t}$ **[5 marks]**

6. (a) Define a Fundamental set. **[1 mark]**
   (b) Show that $z = y_1.y_2$ is a solution of $z''' + 4a(x)z' + 2a'(x)z = 0$ if y_1 and y_2 are two solutions of $y'' + a(x)y = 0$. Also show that if $\{\phi_1, \phi_2\}$ forms a fundamental set of $y'' + a(x)y = 0$, then $\{\phi_1^2, \phi_1\phi_2, \phi_2^2\}$ forms a fundamental set of $z''' + 4a(x)z' + 2a'(x)z = 0$. **[9 marks]**

7. (a) Write down the conditions for two polynomial operators to be equal? **[2 marks]**

(b) Show that for any two polynomial operators P(D) and Q(D),
   i. [P(D)+Q(D)]u=P(D)u+Q(D) u
   ii. [P(D) Q(D)]u=P(D)u Q(D) u **[8 marks]**

8. (a) From a chemical analysis it was determined that the residual amount of C-14 present in the samples of a charcoal taken from a cave under study was 15% of the original amount. Given that the half life of C-14 is 5600 years and that the quantity of C-14 in the sample satisfies the decay equation, then
   a) Find the decay constant k
   b) Find Q(t), the quantity of C-14 at any time t if the initial amount is $Q(0) = Q_o$.
   c) Find the age of the charcoal remains and hence the approximate age of the cave. **[5 marks]**

   (b) Check if the equation $(3xy^3 + 2y)dx + 2x^2y^2dy$ is exact or not. If not, make it exact and find the solution. **[5 marks]**

9. (a) Find the solution of $\dfrac{d^2y}{dt^2} + 4y = 0$ **[5 marks]**

   (b) Discuss the existence and uniqueness theorem for the initial valued problem $\dfrac{dy}{dx} = x + y$ with $y(0) = 1$ and the domain $D : |x| \leq 1, |y - 1| \leq 1$ and hence find the solution. **[5 marks]**

10. (a) Define the critical point for an autonomous system of differential equations. Find the critical points of $\dfrac{d^2x}{dt^2} + \dfrac{dx}{dt} + (2x - x^2 - x^3) = 0$ **[5 marks]**

   (b) Determine the type and stability of the critical point of $(0, 0)$ of the non linear system of equation $\dfrac{dx}{dt} = 8x - y^2, \dfrac{dy}{dt} = -6y + 6x^2$. **[5 marks]**

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