Date:

Registration number:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27
M.Sc. PHYSICS - I SEMESTER
SEMESTER EXAMINATION: OCTOBER 2021
(Examination conducted in January-March 2022)

PH 7220/ 7221 – MATHEMATICAL PHYSICS

Time- 2 ½ hrs Max Marks-70

This question paper contains Two printed pages and Two parts

Part A
Answer any FIVE questions. Each question carries 10 marks

[5 x 10 = 50]

1. Let C be the contour $z = 3e^{i\theta}, 0 \leq \theta \leq \pi$. Show that $\left|\int_C \frac{z^{1/2}}{(z^2+1)} dz\right| \leq 3\sqrt{3}\pi/8$. Assume that $|\int f(z)dz| \leq |f(z)|.L$ where L is the length of the contour. **[10]**
2. (a). Find the residues of $f(z) = (z^2 - 2z)/[(z+1)^2(z^2+4)]$

   (b). Let C be the boundary of the square whose sides lie along the lines x=$\pm 2, y = \pm 2$ where C is described in the positive sense. Evaluate: $(i). \oint e^{-z} dz/(z - \pi i/2)$ and $(ii). \oint Cos\, z\, dz/[z(z^2 + 8)]$. **[5+5]**
3. (a) Find the Fourier Transform of Exponential decay ($e^{-x/a}$). Show that the power spectrum of it has a Lorentzian profile. What is the physical significance?

   (b) Find the Fourier Transform of two delta functions spaced equally on either side of the origin. **[5+5]**
4. A covariant tensor has components $xy, 2y - z^2, xz$ in rectangular co-ordinates. Find its covariant components in spherical co-ordinates. **[10]**
5. (a). Starting from the generating function for Bessel's Function $J_n(x)$, find the Jacobi series and hence show that,

   $$(i). cosx = J_0 - 2J_2 + 2J_4 - \cdots , \qquad (ii). sinx = 2J_1 - 2J_3 + 2J_5 - \cdots$$

   (b). Using Rodrigue's Formula: $P(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$. Find the following Legendre polynomial and plot the functions.

   $(i).\ P_0(x), (ii),\ P_1(x), (iii).\ P_2(x), (iv).\ P_3(x)$. **[5+5]**
6. (a). With a suitable example, explain the properties of SU(2) group.

   (b). Derive an expression for the one-dimensional heat flow. **[5+5]**
7. Using the method of separation of variables, obtain the solution of the wave equation for $(i). k = 0, (ii). k > 0, (iii). k < 0$.

   $$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}.$$ **[10]**

Part B

Answer any Four questions. Each question carries 5 marks

[4 x5 = 20]

8. Prove that, $J_{-n}(x) = (-1)^n J_n(x)$ using Bessel Polynomials.
9. Express the function $H(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Hermite Polynomial.

   $[H_0(x) = 1, H_1(x) = 2x,\ H_2(x) = 4x^2 - 2,\ H_3(x) = 8x^3 - 12x, H_4(x) = 16x^4 - 48x^2 + 12]$
10. State and prove convolution theorem.
11. Prove Parseval's identity.
12. Find the Fourier Series decomposition of a rectangular wave form having a width a. What do you understand by Gibb's phenomenon?
13. (a). Form a partial differential equation by eliminating arbitrary function:

$$Z = f(x^2 - y^2)$$

   (b). Solve the partial differential equation using Lagrange's method $(Pp + Qq = R)$:

$$y^2 p - xyq = x(z - 2y)$$

[2+3]